AUTOMATED DOT MAPPING - HOW TO GENERATE DOT CLUSTERS

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ABSTRACT

Dot maps are used to visualize absolute values, such as population figures or other amounts. Especially dot distribution maps can serve for recognizing spatial distribution patterns. While other cartographic representation methods are already automated, e.g. choropleth maps, dot mapping is still limited to dot density maps. This paper presents a method of generating dot clusters in dot distribution maps. The difference to dot density maps is the centered placement of the dots. The developed technique unites the advantages of regular and random dot placement by guaranteeing a certain number of dots and pseudo-random dot placement to avoid regular structures.

BACKGROUND AND OBJECTIVES

For a long time the production of maps was in the hands of cartographers. Without knowledge about the analogue mapping techniques it was impossible to produce high-quality maps. Nowadays the situation has changed. Many cartographic representation methods have been automated, giving even laymen the chance to produce good maps. But there are still some map types that are not yet transferred to the digital age. Dot distribution maps belong to these. To bridge this gap a new approach was developed. The whole method covers all steps of producing a dot map from determining the size, value and number of dots to actually placing the dots in the map. Only the aspect of calculating the positions of the dots will be covered in this paper.

There have been some works on automated dot placement in dot maps in the past. Aschenbrenner (1989) develops an algorithm for regular dot patterns, while Ditz (2000) and Kimerling (2009) focus on random dot placement.

APPROACH AND METHODS

First the number of dots that can be placed in the map is determined. Therefore a circle around each reference center of an amount, e.g. a settlement for population figures, is drawn. The circle reaches its maximum size when it touches the circle of an adjacent reference center. This circle determines the place that can be used to place dots. The placement of dots bases on a spiral whirl like a spiral galaxy where the stars represent the dots. The basic form of one spiral of this whirl is a logarithmic spiral. The mathematical description of the logarithmic spiral bases on Euler's constant *e*. Besides Cartesian coordinates polar coordinates can be used to describe the spiral as well. Polar coordinates describe the coordinates against the azimuth direction angle φ.

Polar coordinates: $r = e^{a \cdot \varphi}$

Cartesian coordinates: $x = e^{a \cdot \varphi} \cdot \cos \varphi$ $\qquad$ $y = e^{a \cdot \varphi} \cdot \sin \varphi$

whereupon in each case is applied: *a > 0*

To guarantee compliance with the minimal distance between dots the parameter *R* is introduced. It can be understood as half the irreducible distance. It is composed by the radius r_P of a dot and half the minimal distance between two dots (see figure 1). For the minimal distance Koch (2002) proposes a value from 0.15 to 0.25 mm. For further considerations the average – 0.2mm – was chosen. So *R* is the sum of the dot radius and 0.1mm.

$$\overline{P_i P_{i+1}} \geq 2 \cdot R$$
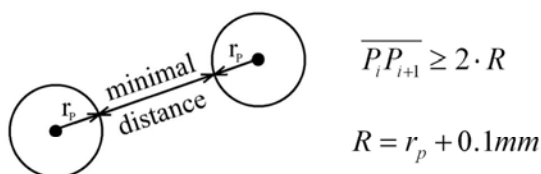
$$R = r_p + 0.1mm$$

Figure 1: Parameter R = half the irreducible distance

Koch/Stams (2002) mention that for dot maps the dot diameter usually ranges from 0.3 mm to 1 mm. That defines the domain of the parameter *R*:

$$R_{min} = 0.15mm + 0.1mm = 0.25mm \qquad\qquad R_{max} = 0.5mm + 0.1mm = 0.6mm$$

Due to restrictions in keeping a minimal distance between the dots a maximum of six homogeneous basic spirals (identical parameter *a*) can build a spiral whirl by being rotated around the reference center. Each single spiral will be referred to as 'spiral arm' in doing so the described basic spirals will be called 'main arms'. A number of less than four main arms is considered as not reasonable. The dots of the dot cluster are placed along the spiral arms. The exact position of dots is determined by the intersection of concentric circles around the reference center with the spiral arms. The radii of these concentric circles depend on the size of dots and are calculated as follows: $n \cdot 2R$ ($n \in N, \ n \geq 1$). The angle $\Delta\varphi$ that determines the rotation of the spiral arms depends on the number *z* of spiral arms.

$$\Delta\varphi = \frac{2 \cdot \pi}{z}$$

The first dot of the dot cluster is the origin. The position of the following dots depends on the size of dots and the according irreducible distance 2*R*. The azimuth direction angle φ unfolds from the condition that dots are placed where concentric circles (radius $r = i \cdot 2R$) intersect the spiral arm. For $R \geq 0.5$ this results in:

$$\varphi_i = \frac{1}{a} \cdot \ln(i \cdot 2R) \quad \text{with } i \in N, \ i \geq 1$$

Due to the fact, that a logarithmic spiral always starts with the point $P_0(1,0)$ the case of *R* < 0.5 needs special treatment in calculating the first dot coordinates. None of the dots along the spiral is located at the origin. The dot cluster composed by dots along the spiral arms therefore has a hole in its centre, which is unwanted. According to the size of dots and along with that depending on the parameter *R* there are three different scenarios of placing the first dots (see figure 2).
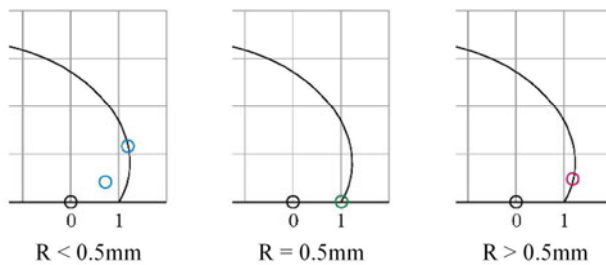

Figure 2: Constellation of the first dots depending on R

The first dot of the dot cluster is always the origin. If half the irreducible distance *R* is less than 0.5 mm, there will be an intermediate dot between the origin and the first dot along the spiral. If *R* is exactly 0.5 mm the first dot along the spiral will be its starting point. And if *R* is greater than 0.5 mm the first dot along the spiral will accordingly be placed farer away from the start point of the spiral.

With the calculated azimuth direction angles the dot coordinates can be calculated in Cartesian coordinates as well as in polar coordinates. The basic calculations are identical for all spiral arms with those of the first one shown above. The single difference is, that dots are rotated by a certain angle. To determine the coordinates of a dot it is essential to take into account whether the dot is located along a main arm or along a secondary arm. Secondary arms are spiral arms whose parameter *a* is identical to the one of main arms. They are placed in the middle between already existing spiral arms to compose a rather compact dot cluster. But these secondary arms are populated with dots starting at a larger distance from the origin. According to the wanted quantity of dots this process of introducing secondary arms can be undergone several times. This, the secondary arms are subdivided into different stages of extension. With every new stage the number of newly added spiral arms and the total number of spiral arms duplicates. There will be further explanations later.

For dots on main arms is defined:

z : quantity of main arms, $z \in N, \ z \in [4;6]$

n : number of spiral arm, $n \in \mathrm{N}$, $n \in [1; z]$

i : index of dot, $i \in \mathrm{N}$, $i \geq 0$

$$x_i = i \cdot 2R \cdot \cos\left(\varphi_i + (n-1) \cdot \frac{2\pi}{z}\right)$$

$$y_i = i \cdot 2R \cdot \sin\left(\varphi_i + (n-1) \cdot \frac{2\pi}{z}\right)$$

To calculate dots on secondary arms:

z : Quantity of main arms, $z \in \mathrm{N}$, $z \in [4; 6]$

t : Stage of extension of the secondary arm, $t \in \mathrm{N}$, $t > 0$

n : Number of spiral arm, $n \in \mathrm{N}$, $n \in [1; 2^{(t-1)} \cdot z]$

i : index of dot, $i \in \mathrm{N}$, $i \geq 0$

A dot on the first secondary arm of a certain stage of extension is rotated compared to the first main arm by the angle $\lambda = \alpha / 2^t$. With $\alpha = 2\pi / z$ this leads to: $\lambda = \pi / \left(2^{(t-1)} \cdot z\right)$. This 'basic rotation angle' is identical for all dots on all secondary arms of this stage. The rotation angle δ to reach the other secondary arms of this stage is a multiple of twice the 'basic rotation angle'.

$$\delta = 2\lambda = \frac{\pi}{2^{(t-2)} \cdot z}$$

Coordinates are computed based on:

$$x_i = i \cdot 2R \cdot \cos(\varphi_i + \lambda + (n-1) \cdot \delta) = 2R \cdot \cos\left(\varphi_i + \frac{\pi}{2^{(t-1)} \cdot z} + (n-1) \cdot \frac{\pi}{2^{(t-2)} \cdot z}\right)$$

$$y_i = i \cdot 2R \cdot \sin(\varphi_i + \lambda + (n-1) \cdot \delta) = 2R \cdot \sin\left(\varphi_i + \frac{\pi}{2^{(t-1)} \cdot z} + (n-1) \cdot \frac{\pi}{2^{(t-2)} \cdot z}\right)$$

For more details on the spiral approach, especially regarding the order of placing the dots, see Hey (2010). Figure 3 shows a dot cluster formed by five spiral arms. But the more dots are placed the more obvious becomes the spiral structure. Therefore some more spiral arms, so-called secondary arms, are introduced. They are placed right in the middle between already existing spiral arms, but they are populated with dots only when the distance to adjacent arms is big enough. The process of introducing new spiral arms can be repeated leading to a larger spiral whirl (see figure 4).
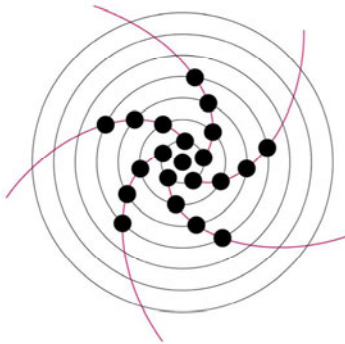

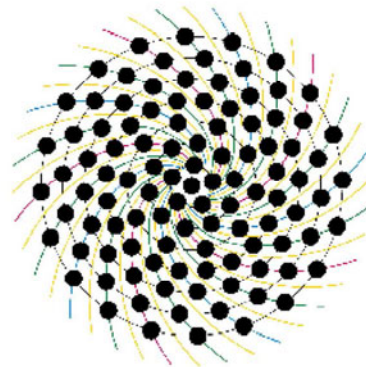Figure 3: Dot placement in a dot cluster of only 5 spiral arms

Figure 4: Dot placement in a dot cluster with additional spiral arms

This method of dot placement leads to very regular dot patterns. Hand-drawn dot distribution maps do not have this regular look. So the symmetry of the dot cluster has to be blurred. Therefore the calculated dot positions are slightly shifted. To avoid overlapping the space in which the dots are moved has to be limited. The problem is solved by calculating dot positions for larger dots than shall actually appear in the map. These larger dots define the space in which the smaller dots, which will appear in the map, can be moved. Figure 5 shows the theoretic model. The difference in size is enlarged to clarify the principle. The

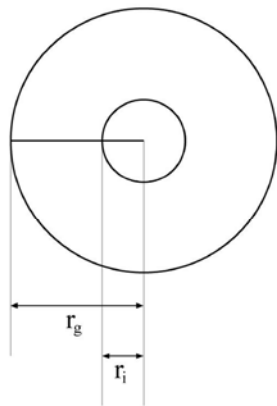dashed dots in figure 6 symbolize possible positions of the actual dot in the space provided by the larger dot.
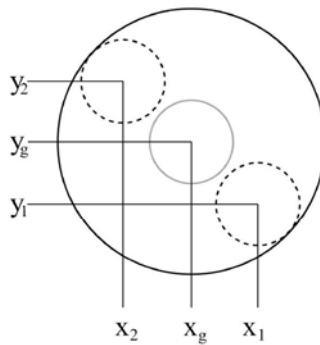

Figure 5: Enlarged r_g and actual r_i dot   Figure 6: Possible dot positions

The shift is calculated based on polar coordinates, which describe coordinates against the azimuth direction angle. This angle is created by using a random number generator. The seed used for that is a coordinate of the original dot position. This guarantees individual random numbers for each dot but also the same random number for each dot when created a second time. To reach the maximum effect of blurring the symmetry of the dot cluster the distance used for the polar coordinates is the maximum possible distance: $dist_{max} = r_g - r_i$.

RESULTS

The presented method was tested with a small data set of three reference centers. With these reference centers the method of determining the space available for placing the dots of the dot cluster was tested and then the dot positions have been calculated. When the space available for dot placement is known the number of dots with a given size can be calculated. This number is then used to calculate the dot positions and it can also be used to find a suitable dot value. As this paper deals with creating the dot cluster the finding of the dot value will not be considered here. The calculation of the dot positions uses several steps. First the dot positions of the enlarged dots are calculated. Figure 7 shows the dots of both sizes in their original position.
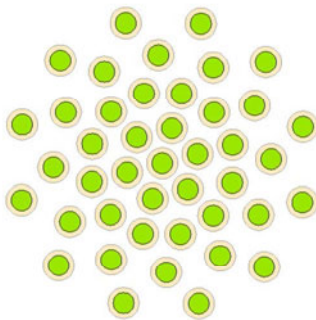

Figure 7: Enlarged and actual dots

Then the original dots are shifted in a pseudo-random way to blur the symmetry of the dot cluster (see figure 8). The resulting dot cluster is recognizable as connected dot cluster but its symmetry is no that obvious any more (see figure 9).
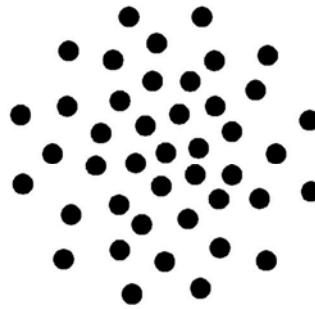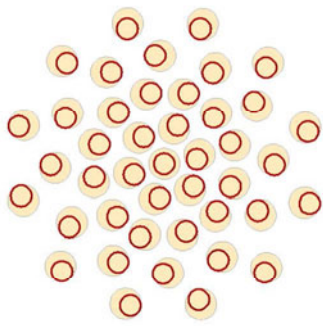
Figure 8: Enlarged dots and shifted actual dots   Figure 9: Resulting dot cluster

The number of dots composing the dot cluster depends on the amount that will be mapped at this reference center. Due to the specific order in placing the dots (see Hey 2010) it is guaranteed that the dot cluster will grow well balanced to each direction.

As the presented method has only been applied to a small test data set there are no results in comparing this method to the classic hand drawn maps yet. It is assumed that the time saving aspect will be an advantage of this method, while it cannot be said if the appearance of the dot clusters will reach the quality of classic dot distribution maps.

CONCLUSION AND FUTURE PLANS

The tests showed that it is possible to create dot clusters completely automatically. The pseudo-random dot placement algorithm produces the same output at all times but individually for each dot cluster. So the resulting dot distribution map will look the same when the mapped data and the scale remain unchanged. This is important to avoid changes in the map that do not result from changes in the data but from a different production date of the map. Dot clusters having equal numbers of dots will not look identically (see figure 10). This contributes to a more natural look of the map concealing the automatic production.

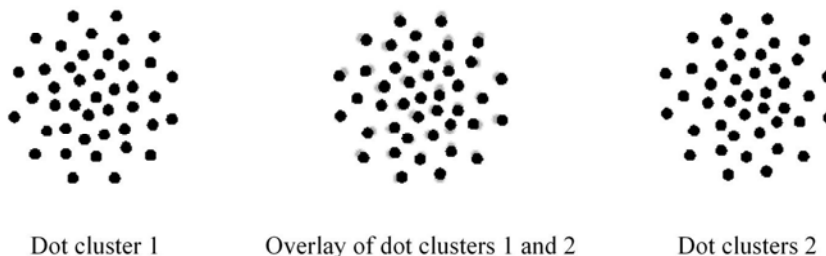Dot cluster 1   Overlay of dot clusters 1 and 2   Dot clusters 2

Figure 10: Resulting dot clusters of the same size but of different reference centers

The presented part of an algorithm to automatically generate dot distribution maps will be integrated in a software tool that will guide all kinds of users through the process of generating good dot distribution maps even without cartographic knowledge. For cartographers it offers the advantage of simplifying and speeding up the process of map design. There will be further research on how this methods works with larger data sets and in comparing the automatically produced dot distributions to those of the classic hand-drawn dot maps.

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