NEW CONCLUSIONS ABOUT THE MATHEMATICAL FOUNDATIONS OF LA COSA'S WORLD MAP USING COMPUTER-ASSISTED METHODS.

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Abstract

The aim of this paper is to present the results of a mathematical analysis of La Cosa's portolan chart (1500) using general digital systems for drawing, storing, retrieving and representing geographical data. From the mathematical point of view, the main characteristic of this map is the assumption of a spherical world instead of the traditional medieval flat world. For this reason the cartographer has drawn the Equator and Tropic of Cancer lines, giving the map a spherical dimension. The hipothesis defended in this paper is that this trascendental transformation of the conception of the document must imply a radical change of the mathematical relationships in the map.

1 Introduction.

Research into historical cartography has been traditionally focused on the study of map contents and its significance on the development of the world map. However, although the projection is one of the most important problems in cartography, the study of the mathematical basis of historical maps has been neglected, due in part to an incomplete training of researchers which has hindered an in depth investigation of dimensional attributes.

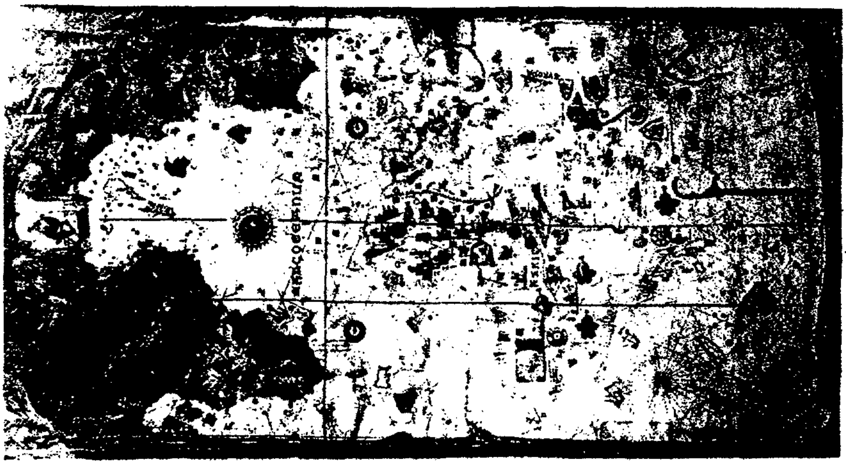
Due to several mathematical uncertainties which exist even to this day, portolan nautical cartography presents an attractive field of work as A. Erik Nordenskiöld [1] had already observed nearly a hundred years ago. He cleared the way for a large number of research projects focusing on the mathematical foundation of portolan charts.

The aim of this paper is to present some of the possibilities offered by new graphic digi-

[1] This paper partially summarizes the result of the research work "The *map as a palimpsest. Juan de la* Cosa's Chart cartographical analysis", financed by Fundación Marcelino Botín of Santander.

tal systems in the analysis of historical maps. Current developments and the computer revolution have opened up a wide range of possibilities for the researcher interested in these kind of issues, though the computer is still hardly being employed in historical cartography research. For this reason, we have concentrated on the study of one of the most outstanding landmarks in the construction of the modern world map: Juan de la Cosa's Chart (1500). This chart combines both undeniable documental value as the first evidence of the Castilian, Portuguese and English discoveries along the American coasts and also interest created by its possible mathematical contributions to the map making technique, as other authors have already pointed out [2, 3].

Figure 1 Juan de la Cosa's Chart.



2 Dimensional uncertainties raised by Portolan cartography.

Portolan charts originated in the 13th century and developed significantly during the 14th and 15th centuries. Unlike other types of maps, these charts are not based on a latitude and longitude (φ, λ) reference system of meridians and parallels, but on that of bearing and distance. This was the refence system essential to the mediterranean navigators in order to set their courses between various ports, and which formed the basic information on the portolan chart.

The method of establishing a bearing and distance point, from a geometrical point of view, corresponds to the polar plane coordinate system which is determined by a linear measure (vector radius) and angular measure. In the case of portolan cartography, the system axis corresponds to the magnetic meridian determined by the compass.

The interest in maintaining the angular value on the chart for its later interpretation with the compass means that we are facing a sort of representation close to a conformal projection but lacking any spherical relation. This similarity to commonly used projections, is one of the reasons why portolan cartography continues to surprise us with its proximity to reality.

The flat chart however, soon began to lose its former validity due to the development of oceanic navigation. In fact, and as Bagrow and Skelton point out [4, p. 118], *"its failure to allow for the convergence of the meridians in higher latitudes and its dependence on compass bearings which took no account of the varying magnetic declination in different parts of the world. These shortcomings had little significance in navigation of European waters, but became serious on (for example) the coasts of the New World"*. Nevertheless, which were the problems which arose on going beyond the old limits and in extending the world map? Moreover, how were these problems resolved?

Juan de la Cosa's Chart is an excellent precedent for the latter questions. Based on the conceptual foundations of the portolan chart, this map shows from a dimensional point of view, the following characteristics:

- it is almost completely drawn from the navigator's empirical information, except the Asian coasts which were derived from Ptolemy,
- Ecuatorial and Tropic of Cancer latitude lines are also drawn,
- it has two graphic scales *(troncos de leguas)* at its upper and lower margins, and
- the geographical and toponymical detail is adequate for an accurate known geographical point identification.

3 A Graphic Data File generation of Juan de la Cosa's Chart.

The conversion of the analogical information contained within the source map into digital or numerical format allows a digital map file to be generated. With this file we can quantify and distribute the map geometrical characteristics, due to its comparison to reality. This operation, traditionally done in a analogical way by the direct overlay of the old and new outlines or point systems, can now be done in a numerical way by measuring coordinates on the digital copy and statistically analysing the results by computer.

By means of this new scaled graphic file, we overcome the problems of the inability to work on the original document (exhibited at Madrid Naval Museum). Also, by means of measurements previously taken from the original chart we can create an arbitrary reference system utilising the bearing network to enable us to geometrically correct the failures of the original support (parchment)[2], and photographical distortions. We are then in a position to analyse the map data with a much higher precision than previously possible by traditional and analogical means. The main advantage of these types of files is that they can be generated without maintaining direct contact with the map and provide direct entry to programmed operation. These techniques allow not only enlargements of the original map to any desired scale, but also combine the dimensional features of the model with the added advantage of photographic detail.

The new image is stored in the computer as organized layers or levels. Each level is associated with a fixed number of elements, and each element can be represented in different graphical ways. As a result, we hold a numerical copy of the original map which can be worked on analitically. This process is based on linear and angular measurement which permit assess to most of the geometric characteristics of the chart such as scale, projection, distortion level, coordinate resolution, instrumentation accuracies, measurement units, graphic scale study, etc.

4 Cartometric analysis: main conclusions.

The comparison of the chart lineal values with true geographical distances is set by the Proportionality Coefficients (PC), which can be expressed as follows:

$$PC = (d / m) \bullet T \bullet 1{,}48 / Gd$$

where d corresponds to the distance value taken from the file in millimeters, m is the value in millimeters of each graphic scale division, T is the graphic scale dimensional value (normally 50 roman miles) and Gd is the geographic distance between the same pair of points. The 1,48 constant will convert our original measure into kilometers. The analysis of the data thus obtained reveals two important characteristics of La Cosa's Chart.

4.1 The presence of a double scale.

When comparing 74 chart distances with true geographic distances, we notice a discrepancy between values in middle latitudes, where the representation should be most accu-

[2] The parchment deformations can be corrected by photogrametrical stereoplotting methods. Its identification can be difficult if we work with a photograph. Only bearing lines can give us an approximate idea of those parts which can be deformed in the source map. Juan de la Cosa's Chart parchment does not show any important deformation, except on the right hand margin where we have not worked.

rate. In fact, taking as a base the value of T=50 roman miles for ecuatorial scales, the coefficients show an average value of 0,841. They are distributed as follows:

Table I Proportionality Coefficients. Juan de la Cosa's Chart.

	Observations	PC	% Error
Mediterranean Sea-Black Sea	35	0,788	21,2
Atlantic Europe	9	0,846	15,4
Atlantic Africa	13	0,994	0,6
America	11	0,817	18,3
General Values [3]	6	0,861	13,9

This systematical imbalance between chart and true geographic distances cannot be due to valuation errors. In our opinion, there is a change of the general 50 Roman-mile graphic scale into another secondary scale of 40 Roman-miles (not drawn), resulting from the application of the cosine trigonometrical function in the 36°N latitude (latitude of the ancient greek *diafragma* which crosses the Mediterranean Sea from Gibraltar to Rhodes)[4]. This is applicable to the chart as a whole except in the Atlantic african coast which shows a clear equivalence with the ecuatorial scale.

When the lack of convergence of the meridians in portolan charts was identified, graphic scales became scales referred to the Equator and charts became oceanic navigation charts. The outcome of the dimensional discrepancy regarding the Equator in the middle latitudes explains why the navigator had to make a change in the ecuatorial module. The solution to this problem was to include other secondary scales in different latitudes which allow measures on small circles to be achieved. The particular scales are in fact conversion graphics of the equatorial degrees in miles, according to the following equivalence:

$$T \bullet \cos\phi = T'$$

where T corresponds to the graphic scale division length in the Equator and T' to its latitude. This implies that the cartographer includes a conversion of itinerary measures depending on the latitude involving a change to the flat chart due to sphericity. The graphic scale change due to latitude is a simple dimensional conversion wich allows us to preserve unchanged the remainder of the chart. Thus, the cartographer resolves part of the distortion generated by the lack of convergence of meridians in an operation which promotes the variable graphic scales of flat charts with latitude grading and also in later charts in the Mercator projection.

3 General values are pair of points wich join not only long distances in the Chart but also measures between the African and South America coasts.

4 This transformation of the equatorial graphic scale is an admitted fact in portolan cartography research [5, 6].

4.2 The use of two different earth modules.

Chart spherity is explained by the relation between itinerary measures and spherical angular values, which vary according to the latitude. The way to express this relationship is the earth module (M), defined as a degree arc length in any great circle over the sphere.

Once we have marked out the distance between the equatorial and tropic of Cancer lines ($\Delta\varphi = 23° 26' 37''$) in our file, we obtain a value for M = 103,6 kms. This general value of the module implies a slight underevaluation of the earth size of 6,82%, although it is smaller than the one given by other authors such as Ptolemy, Columbus or Behaim (Table II).

However, if we draw the parallel system derived from this module value, we can see how the parallels begin gradually to lose their reliability going northwards beyond the Tropic of Cancer (Fig. 2). In our opinion, these important inaccuracies in the Mediterranean Sea are contrary to its well-known position in the 15th and 16th centuries. Then, what can the origin of this latitude disagreement be?

The Berlengas-San Vicente cartographic base was another dimensional control on portolan charts in addition to graphic scales and equatorial and tropical lines. This base set up a 3° arc length from cape San Vicente in its correct position (37°N) to the largest Berlenga island, to the west of cape Sacro in Portugal, and its use in the graduation of charts dates from the 14th century [7].

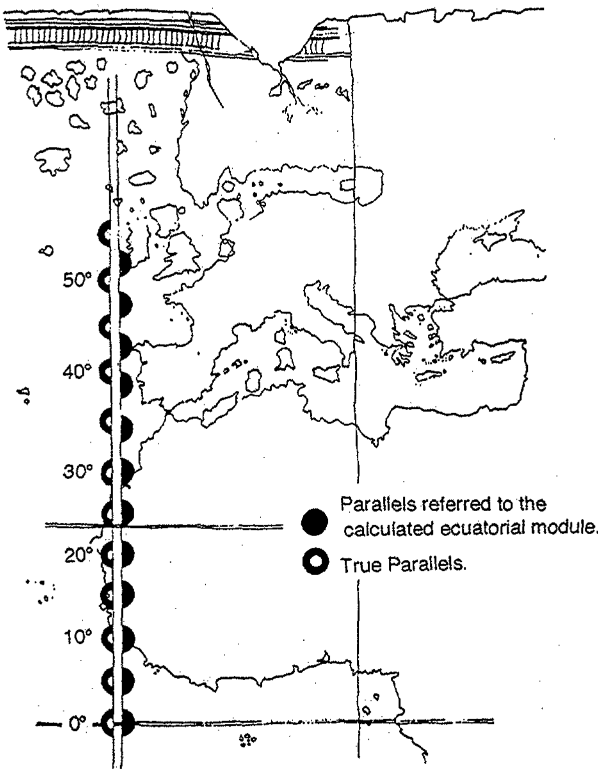
When confirming empirically the use of this cartographic base in La Cosa's Chart, we can distinguish a new module value of 98,66 kms., smaller than the one defined by equinoctial lines. The presence of this new module is most probably due to the union of two previously existing charts since the use of two different earth modules within one single chart by the same cartographer would be most unlikely. Therefore, the presence of two different cartographical foundations joined later in a single document is suggested and, consequently, we can present a new evidence which connects the chart with the later Spanish *Padrón Real.*

For these reasons we can confirm the lack of a projection system in Juan de la Cosa's Chart. Despite all the spherical relations in the chart, other characteristics revealed in our work, such as the lack of geographical coordinates (φ, λ) and of a unique reference system, the presence of two earth modules, or the incidence of magnetic declination of different sign, prevent us from proving the use of a mathematical projection on the chart.

Table II True geographic values and chart values.

	Real values[5]	Juan de la Cosa's Chart
Module	111,19	103,6
Equator	40.030	37.296
Radius	6.371,1	5.935,84

Figure 2 Parallels discrepancy in middle latitudes.



5 We consider a radius value of 6.371,1 kms.

5 Final conclusions.

Our findings prove the presence, in the early years of the XVIth century, of a change in the foundations of portolan cartography. The adaptability of the mathematical basis to the necessities of the new representation cannot be understood without a practical conception perspective on this kind of document. This explains why medieval navigation charts did not come to an end during the Renaissance and played a vital role in the conception of the new world map, as a result of the development of navigation, geography and cartography.

We believe our research has proved the methodological importance of the analysis of dimensional features in historical map research. It has also given us an insight into the making of La Cosa's map. At the same time we point out the important role played by computer systems in graphic information processing in old map studies by means of digital copy making. New documentation trends (scanner digitalization, optical disks, etc.) will permit a very important advance in research into historical cartography. Nevertheless, due to the importance of dimensional data, the generation of a correct cartographic digital file must include, in our opinion, the possibilities offered by terrestrial photogrametrical methods and vector data structures, in the interest of a better positional accuracy.

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