AUTOMATIC MAP GENERALIZATION BASED ON A NEW FRACTAL ANALYSIS METHOD

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Abstract

A new cartographic generalization-oriented method of fractal analysis is proposed in this paper, it provides a quantitative characterization of the cartographic lines and describes the relation between the tolerance values and the features of cartographic lines when the scale changes, which then helps realizing the objectivity、modelization and automation of cartographic generalization process.

1 Introduction

It is well known that cartographic generalization is one of major challenges that GIS and computer-assisted cartography are faced with. In terms of the generalization of cartographic lines, although quite a number of algorithms have been proposed in recent years([3]), the digital encoding of map features and subsequent computer representation at many level of resolution remains a serier of questions; It is essential to preserve the details required for recognizability automatically during map generalization, but what does the recognizability mean? how we describe it digitally? No matter what algorithms are employed, generally speaking, map generalization process and result are affectted distinctly by tolerance values, but what is suitable tolerance value required for representing features objectively? Is it possible to choose reasonable tolerance values automatically according to cartographic line' s graphic feature itself? Cartographic generalization, as the term suggests, serves to generalize the shape or to simplify the complexity of the cartographic lines, but what level of generalization should be required exactly for a known level of scale changing? Can we find out the corresponding digital relation between the level of generalization and the level of scale changing? This paper addresses these questions, proposes a generalization-oriented method of fractal analysis to quantify the features of cartographic lines and to seek the quantitative relation among tolerance values, graphic features and scale changing, thus providing a new way to realize the objectivity, modelization and automation of cartographic line generalization process.

2 Basic concept of fractal geometry

Fractal geometry is a new branch of mathematics that deals with the quantitative description of irregular and fragmented geometrical objects, it exhibits two basic concepts: fractal dimension and self-similarity.

In Euclidean geometry, every curve has a dimension of 1 and every surface has a dimension of 2, dimension is defined as the number of distinct coordinates needed to specify the position of a point in space, however, this definition is not satisfactory for an accurate description of highly irregular and fragmented natural objects. The concept of fractal dimension cuts across the logic of Euclidean geometry, in fractal geometry, dimension is considered as a continuum, the fractal dimension of a curve can be any value between 1 and 2 (and a surface between 2 and 3) according to the complexity of the curve (or surface).

Self-similarity means that the curve (or surface) is made up of copies of itself in a reduced scale. the number of copies(n) and the scale reduction factor(d) can be used to determine the fractal dimension (D), where, $D=\log(n)/\log(d)$ (Falconer, 1990). However, seldom in nature does self-similarity occur and therefore a statistical form of self-similarity is often encountered. Practically, the D value of a curve is estimated by measuring the length of the curve using various step sizes. The more irregular the curve, the greater increase in length as step size decreases; thus D can be calculated for a curve by the regression equation: $\log L=C+k\log d$, and $D=1-k$, where L is the length of the curve; d is the step size; k is the slope of the regression; and C is a constant.

Since the objective of cartographic line generalization is to represent the overall complexity and character of a line at various scales, the fractal analysis could prove to be a very useful method for generalization.

3 Generalization-oriented fractal analysis

It is clear that the fractal dimension describes how the graphic features change with the step size. In cartographic generalization, however, we take a great interest in how the graphic features change with the tolerance values, for this reason, we propose a generalization-oriented estimation method of fractal dimension.

Naturally, investigating the issues of tolerance values should relate to some generalization algorithm. As Douglas-Peucker algorithm is considered as producing the generalization most faithful to the original line and having a most wide use, we might take it as the basis of the new method.

Essentially, the algorithm defines a straight line segment between the first point (called the anchor) and the last point (called the floater) on a line. Perpendiculars between the line segment and each of the original points are then meansured. If in the first iteration (cycle) the length of all perpendiculars is less than the preset tolerance value, the line segment is deemed adequate to represent the line, all points except the anchor and floateer are deleted and the algorithm is terminated. If one of the perpendiculars exceeds the tolerance value, the point before the floater becomes the new floater and the new perpendiculars are measured. this process is repeated until the length of all perpendiculars is found to be less than the preset tolerance value. In this case the line segment is deemed adequate to represent this portion of the line, all intervening points are deleted and the anchor is moved to the floater. A new segment line is now created from the new anchor to the end point in the line (the new floater), the perpendiculars are again measured and the process is repeated. The algorithm continues until the anchor has advanced to the end of the line. Clearly, the level of generalization is directly proportional to the size of the tolerance value. A large tolerance value will produce a highly generalized line.

It is easy to know that the larger the tolerance value, the greater decrease in length of the curve. according to the principle of fractal geometry, we can conclude:

$$L(d) = cd^{1-D} \quad (1)$$

Where L is the length of the curve; d is the value of tolerance; c is a constant, and D is fractal dimension of the curve.

Taking logarithms of equation (1) it yields:

$$\mathrm{Log}L(d) = (1 - D)\mathrm{Log}d + \mathrm{Log}c \quad (2)$$

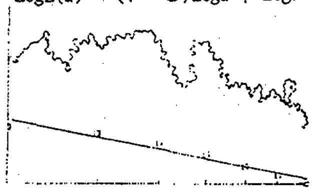


Figure 1: Example of estimation fractal dimension

Obviously, substituting various tolerance values d and corresponding L(d) into equation (2), the value of D can be estimated using a simple linear regression procedure. For example, for the curve shown as figure 1 (a section of coastline in zhejiang province of China, scale 1 : 50000). D=1. 196, r=0. 978(regression coefficient).

Equation (2) provides a generalization-oriented model of fractal analysis, because it describes how D change with d, namely, how the graphicfeatures change with the tolerance values.

4 Principle and algorithm

We can conclude by observing figure 1 that the fractal analysis process for a curve, in fact, have defined a corresponding relation among d, D and L on the statistical meaning, that is to say that we can obtain a corresponding length (a complexity level) of the curve for any tolerance value belonging to the considered range, and vice versa. Now let M_1 and M_2 be the scale denominator of original map and generalized map respectively, d_1 be the tolerance value required for describing original curve, and L_1、L_2 be the length of original curve and generalized curve respectively, then L_2 can be calculated easily by M_1、M_2 and L_1, for example, using the formula proposed by the author in 1993: $L_2=L_1(\frac{M_2}{M_1})^{1-D}$ (where D is fractal dimension of the curve). Substituting L_2 into equation (2), for obtaining the tolerance value d_2 required for describing generalized curve (correspoding to M_2), we then can get:

$$d_2 = e^{\frac{\log L_2-\log C}{1-D}} \tag{3}$$

According to the principle of fractal geometry, the curve described by d_2 has the same fractal dimension with the curve described by d_1 on statistical meaning, this indicates that the generalized curve can preserve the graphic features or characteristics when scale denominators change from M_1 to M_2. Therefore, a new way to produce resonable tolerance values automaticlly according to cartographic line' s graphic feature itself is found out.

Let us summing preceding discussion as following basic algozithm of generalization based on fractal analysis:

(1) Input the point set of original curve: $\{P_i\}_{i=1}^{n}=\{(x_i,y_i)\,|\,i=1,2,\cdots\cdots n\}$;scale denominators: M_1 (correspoding to the original map) and M_2 (correspoding to the generalized map); the point numbers of linear regression: k;

(2) Set $j=1$; $d_j=\min\limits_{i}\left\{\dfrac{|(y_{i+1}-y_{i-1})(x_i-x_{i-1})+(x_{i+1}-x_{i-1})(y_{i-1}-y_i)|}{\sqrt{(x_{i+1}-x_{i-1})^2+(y_{i+1}-y_{i-1})^2}}\right\}$

(3) Perform Douglas-Peuck algorithm for $\{P_i\}_{i=1}^{m}$ taking d_j as tolerance value, then get the remained points: $\{P'_{ij}\}_{i=1}^{m}=\{(x'_{ij},y'_{ij})\,|\,i=1.2,\cdots m\}$;

(4) $L_j=L(d_j)=\sum\limits_{i=1}^{m-1}((x_{ij}-x_{i+1,j})^2+(y_{ij}-y_{i+1,j})^2)^{\frac{1}{2}}$;

(5) $j+1\rightarrow j, d'_1\rightarrow d_j$, Repeat (3)~(4) until $j>k$;

(6) $B=\dfrac{\sum\limits_{j=1}^{k}\log L(d_j)\cdot\log(d_j)-\frac{1}{k}\left(\sum\limits_{j=1}^{k}\log L(d_j)\right)\left(\sum\limits_{j=1}^{k}\log(d_j)\right)}{\sum\limits_{j=1}^{k}(\log L(d_j))^2-\frac{1}{k}\left(\sum\limits_{j=1}^{k}\log L(d_j)\right)^2}$;

$A=\frac{1}{k}\sum\limits_{j=1}^{k}\log L(d_j)-\frac{B}{k}\sum\limits_{j=1}^{k}\log L(d_j)$; $L_2=L_1\left(\frac{M_2}{M_1}\right)^B$; $d'_2=e^{\frac{\log L_2-A}{B}}$

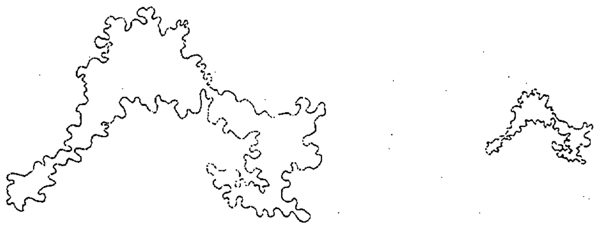
(7) Perform Douglas-Peuck algorithm for $\{P_i\}_{i=1}^{n}$ taking d'_2 as tolerance value, and output generalized curve, which described by the remained points.

5 Practical example

For the given original curve (a coastline of Touman island in zhejiang province of China, scale: 1 : 5000), shown as figure 2(a), using the proposed method, we conduct fractal analysis and get the following result:

M_1	M_2	range of self-similarity	L_1(mm)	L_2(mm)	d_1(mm)	d_2(mm)	D	r
50,000	200,000	[0, 4.44]	21556.15	20647.97	6.14	30.089	1.61	0.985

The corresponding test result of automatically generalization based on fractal analysis is shown in figure 2(b).



(a) (b)

Figure 2: A test for coastline generalization

6 Conclusions

None of the line generalization algorithms published in the literature appears to preserve with full certainly both self-similarity and fractal dimension (Muller, 1989). It is encouraging that the proposed algorithm based on new fractal analysis method eliminates this sorriness and makes it becoming reality to produce suitabe tolerance value automatically according to curve' s graphic features itself. The results of this study have indicated that the fractal analysis can become another potential tool in the cartographic process.

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