INTERPOLATION ADJUSTMENT OF MAP BY THE METHOD OF FINITE ELEMENTS

Dr. Eng. Nguyen Duc Minh
Cartographic Public Haus
General Department of Land Administration
Lang Trung Str. Dist Dong Da Hanoi Vietnam

One of the key objectives of our International Cartographic Association (ICA) is disseminating and applying advanced technologies in mapping over the world. In developed countries the equipment production, technology and the mapping process have been automated. However in under-developed countries, although their teritories are large and the resources are potentially abundant but they have not enough conditions to establish their own maps.

Moreover, the software on GIS and mapping are also required to be improved to overcome its disadvantage.

In cartography, three following problems are very important : position, content and form of representation of the objects, of which the position is of a key importance. For instance, the position (location) of rivers, roads, cities... The contents of the objects can be updated through information collection and treated, for example, by the method of statistic analysis.And the form of representation depends on the human aesthetic skills.

The purpose of this paper is to recall the application of the concept of the method of finite elements. Thanks to this method, we can determine position of the objects on the map more efficiently.

The method of finite elements will be given in the reference. However, we point out first some simple problems but often used in cartography :

1. Transformation of co-ordinates

General formulae in the 2-dimension coordinate system :

$$X = Xo + u\cos(A)\,q - v\sin(A)\,q \quad (1)$$

$$Y = Yo + v\cos(A)\,q + u\sin(A)\,q \quad (2)$$

A is rotary angle and q is scale.

(u, v) are co-ordinates in the original system.

(X, Y) are co-ordinates in the destination system.

If the origin (Xo, Yo) is known, it requires at least 2 known points. On the contrary, it requires 3 given points. If the given points are in excess of 3 points, the method of least squares can be used.

For persons without high skills in method of calculation and programming, simple method can be used. The results obtained are similar.

Suppose we have to transform and reduce to the same scale from system (ξ,) into system (X,Y) with the number of given points more than 3.

$$X = Xo + (\xi \cos \phi - \eta \sin \phi)\, q, \tag{3}$$

$$Y = Yo + (\eta \cos \phi + \xi \sin \phi)\, q. \tag{4}$$

Where

$$tg\,\alpha_{ij} = \frac{\Delta Yij}{\Delta Xij}; \qquad tg\,\psi = \frac{\Delta \eta i,j}{\Delta \xi i,j}; \tag{5}$$

$$S_{ij} = \sqrt{\Delta X^2 ij + \Delta Y^2 ij}\ ; \quad S'_{ij} = \sqrt{\Delta \xi^2 ij + \Delta \eta^2 ij} \tag{6}$$

$$\phi_{ij} = \alpha_{ij} - \psi_{ij}\ ; \qquad q_{ij} = \frac{S_{ij}}{S'_{ij}} \tag{7}$$

$$\phi = \frac{p_{12}\phi 12 + p_{13}\phi 13 + \ldots + p_{23}\phi 23 + p_{24}\phi 24 + \ldots}{p_{12} + p_{13} + \ldots + p_{23}\, p_{24} + \ldots} \tag{8}$$

$$q = \frac{p_{12}\, q_{12} + p_{13}\, q_{13} + \ldots + p_{23}\, q_{23} + p_{24}\, q_{24} + \ldots}{p_{12} + p_{13} + \ldots + p_{23} + p_{24} + \ldots} \tag{9}$$

Where Pij is the weight which can be chosen as the distance between point i and point j.

2 . Interpolation

In interpolation, we may use various polynoms together with the method of least squares, spline function, etc...

The persons without high skills can use the method of average weight which gives the same results as the above mentioned methods.

The formula are as follows : ·

$$Z_M = \frac{\sum Z_{LM} P_{LM}}{\sum P_{LM}},$$

P_{LM} is the respective weight possibly chosen in once of the following values depending on different purposes :

$$P^1_{LM} = \frac{1}{S_{LM}} \; ; \; P^2_{LM} = \frac{1}{S^2_{LM}} \; ; \; P^3_{LM} = \frac{1}{\sqrt{S_{LM}}} \cdot$$

In our opinion, for interpolation in cartography, it is desirable to use P^2_{LM}, S is the distance between interpolated point M and the given point L.

For instance, the transformation of plane coordinates of a function Z, can be applied to each X, Y, B, L or can be used for interpolating the altitude, etc...

The scope of applying transformation and interpolation can be used to the whole mapping area or any part thereof.

3 . Transformation and interpolation in cartography using the method of finite elements.

Now let us go to the application of the method of finite elements in cartography.

So far, there are some restrictions available in the software on GIS and AUTOCAD to reflect the characteristic features of cartography.

For instance, when smoothing the curve, either the trajectory or the given points cannot be fixed on the curve (for spline) or if going through the given points, they can not keep the features of the relief. For example, the river can be located on the slope, or the road is distorted as the result of drawing.

To overcome this problem, the digitalization must be made by using a very large number of points and between two consecutive points must be straight segment. But in this way, the volume of work is too much increasing and so is the volume of stored information.

Our aim is to control the trajectory and the surface as well. The mathematical properties of a curve (or surface) which require to be ensured such as : smoothness (existence of derivatives to a required degree) and the function must be continuous, especially for the ground surface which is very complex.

If the concept of finite elements method particular interpolation are not applied, the proper description can not be achievable. So, we propose applying the results of demonstrating the theoreme of Berschtein, that is the method Berschtein in interpolating the curves and surfaces.

Interpolating results for the lines in cartography must meet following requirements :

(a). The interpolated lines must go through every given point and keep their features.

(b). Maximum distances between arc and its segment ds (fig. 1) must be controlled.

(c). The basic mathematical conditions of the function must be maintained (smooth and continuous).

All the requirements can be meet thanks to the application of Bezier lines (fig. 1).

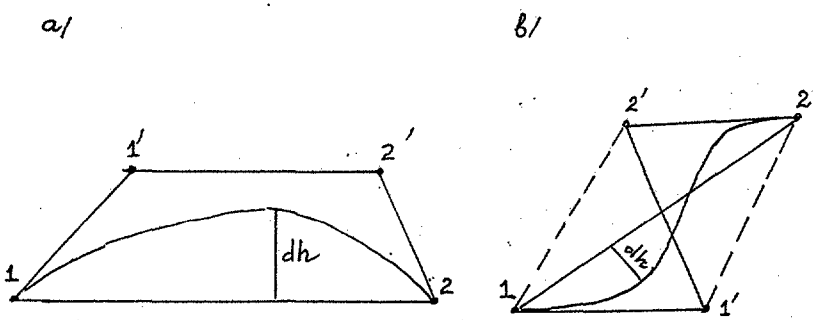

Figure 1: Example of a line Bezier

The theoreme Berschtein which was demonstrated in the early 20th century, found its application late this century by Bezier [2], [3].

Different from polynomial or spline interpolation is the interpolation in the frame Bezier (in the Bezer polygone). Thanks to Bezer frame (polygone), we can control the line (surface) according to the user's demand. For instance, once of these demands is that the contour lines must not intersect themselves, the road or river must maintain their true positions. See [1], [2], [3].

The application of Bezier lines of surfaces in surveying and mapping was proposed by the author in the Bulgarian review Geodesy and Cartography (May 1986). It has been applied in practice to interpolate the contour lines of the other authors (February 1987).

The interpolation for elements of triangle or other figures has been described in [1], [3], [4], [5].

The interpolation can be made better by Bezier polynome itself and can be also made through other methods but in the space of interpolation which is Bezier frame.

We think that there is no need to describe again the method of interpolation, which can be found in the part of reference.

Conclusions and recommendation :

For the cartographers, we think that this earth belong to our common ownership.

The territories of Africa or Asia are also requiring various kinds of map and information on environment and natural resources. In these countries they also are able establish map for themselves with new knowledge.

With common computers or PC in-hand, the cartographers can calculate or establish simple programs according to the above - said ideas to improve efficiency and quality.

In addition, specialists also need to create software serving GIS and mapping to meet the specific requirements of cartography. We hope that our said opinions may be useful for the 17th conference of ICA.

Reference

[1] Adams J.A., 1975. The Intrinsic Method for curve Definition, Computer Aided Design, 7. 4, 243 - 249.

[2] Bezier P., 1968., How Renault use Numerical Control for car body design and tooling, SEA paper 680010.

[3] Béziner P., 1972. Numerical Control: Mathematic and Application, Wiley.

[4] Faux, B. 1978. Computational geometry for Design and Manufactura, Wiley.

[5] Mitchell. A., 1977. The Finite Element method in partial Differential equation, Wiley.