THE REPRODUCTION OF TERRAIN MODELS: FROM THE GEOMETRICAL ARTS TO THE GEOMORPHOLOGICAL SCIENCE

Masaharu Ishii
Research Section, Alps Mapping Co.,Ltd.
2-24 Shochiku-cho, Chikusa-ku Nagoya
Aichi 464, Japan

Abstract

We present a novel method to give a digital terrain model from contour data. The method is constituted on a boundary value problem for the non-linear operator which expresses geomorphological features. In this result, the information of contours is fully used and a fine realistic model with physical consistency is given. We mention the framework and discuss the features given by the operator and show the outputs obtained by the method and by others to compare.

1 Introduction

Here we present a novel solution of the problem to give a digital terrain model from contour data. The solution is constituted on a flexible framework and based on geomorphological consideration. In this result, the information of contour data is fully used in the computation and a fine realistic model with physical consistency is given. The terrain models do not only become free from the unnatural noises which are unavoidable by the conventional methods, but also have the fractal feature of landform.

For a long time, the problem has been merely considered as one of mathematical interpolations. Hence, from the applied mathematics, various solutions were transferred; i) the average of the spline curves in different directions [1], ii) one using the triangular decomposition [2,3], iii) the spline surface, iv) the minimal-energy surface [4], v) the low-pass filter [5], vi) one which mixes above methods in a certain manner, ..., etc. Such conventional methods bring various strangenesses into the landforms. For example, i) generates huge artificial ditches and steps or the starlike noise, ii) leaves texture of spanning triangles, iii),iv) and v) give funny smooth surfaces like bended plastics and erase the lines of valleys and ridges. ...., etc. All of them lack in common the geomorphological features such as the miniute folds and wrinkles.

It seems that the efforts of the most studies today (for example [7]) are made towards the suppression of the strangenesses within the above frameworks. However, the studies can succeed only partially. The essential cause for the strangenesses is that all of the methods are merely geometrical arts ignoring the physical features of landform.

To obtain natural terrain models, we must consider this problem as the physical reproduction of landform from the contour data and must base the theory of the reproduction on geomorphological science. We must notice that the problem is the addition of information. A terrain model must be regarded as a synthesis of some information and the contour data.

Hence, we divide the problem into two concepts; how we add the information and what information we add. The former is the framework of the addition and the latter is the content of the addition. In the conventional methods, the separation of the two were not considerd, and their frameworks were not constituted so as to separate from their additional information. Accordingly in the most cases, *their frameworks dominate the landforms by forcing only their* own features. The contents of the addition have no geomorphological consistency. Therefore the various strangenesses appear.

For example, the framework of iv) gives the minimality of the strain energy of the surface, and that of v) gives the additional information that landform allows only folds larger than the maximum distance between contours. The former has no geomorphological sharpness and the latter denies the fractal feature. The additonal information must be restricted within the physical necessity.

The reproduction problem is how we add the general information of the geomorphological features to the particular information of contour data. However, it is almost impossible to express such features in the conventional frameworks. We need to prepare a framework with the capacity enough for the expression. Here we introduce a boundary value problem for a non-linear operator as the mathematical framework to make this addition possible. The contour lines and the elevational values are regarded as the boundaries and the boundary values, and the geomorphological features are expressed by the operator. We can handle additional information widely by selecting the non-linear operator in this framework, and translate the geomorphological requirements for the landform into the mathematical conditions for the operator. In the mathematical studies of erosion, boundary value problems were discussed [7], but the operators used there were linear and the geomorphological features were not satisfactorily expressed. Only a non-linear operator makes it possible.

We give a formulation and a solution of the framework in section 2, and discuss the additional geomorphological contents given by the operator in section 3, and show the shadings of terrain models obtaind by this method and others in section 4.

2 Mathematical framework

Here we show the mathematical framework of the new method in 2.1, and give a solution in 2.2.

2.1 Formulation of framework

We prepare the terminology for the formulation. Let L be a Banach space of real functions on a real 2-dimensional domain D, $||\cdot||$ a norm of this space, $\mathrm{Hom}(L)$ the whole bounded operator mapping from L into L, C_i ($\subset D$, $i = 1, 2, \cdots$) the contour lines, and v_i ($i = 1, 2, \cdots$) the corresponding elevational values. An element $h(x, y)$ of L expresses the elevational values on D, namely the terrain model.

Assume that the equation dominating landforms exists and it is expressed by

$$h = T\,h \qquad (\,h \in L,\ T \in \mathrm{Hom}(L)\,) \tag{1}$$

with the boundary value condition

$$^{\forall}(x, y) \in C_i\ (\subset D): \quad h(x, y) = v_i \qquad (i = 1, 2, \cdots). \tag{2}$$

The formulation denotes two qualitatively different informations. The one given by T is general and the other given by the contour data is particular. They are indispensable to the reproduction problem. If the operator expresses the general geomorophological features, a natural landfrom corresponding to the boundary condition is obtained.

We discuss the requirements for the operator in section 3. Note that we must discretize the domain D to code the formulation in computer.

2.2 Solution in framework

To solve the boundary value problem of (1) and (2), we operate T iteratively on an initial point h_0 ($\in L$). Then the solution is given by

$$h = \lim_{N \to \infty} T^N h_0. \tag{3}$$

This limit converges if

$$\| \dot{T}(h_0) \| < 1, \tag{4}$$

where $\dot{T}$ is the Fréchet differential of T. T may be modified a little in every iteration in order to converge and, if any, to satisfy other conditions.

In the case of a non-linear operator, the problem can only be solved by iterative method. Then the convergence condition must be satisfied. We take the stacking model as the initial point for the simplicity. Hence from now on, we consider the condition only at the initial point.

3 Contents given by operator

Here we give the limit of the framework and discuss the geomorphological content given by the operator.

3.1 Limit of framework

We pick up important geomorphological features and make the model. Of course such features are restricted within what can be expressed as the fixed point of the operator. Furthermore, the operator expressing the features must satisfy the convergence condition. The limit of the framework is determined from the two restrictions. The influence of the mathematical limit on the geomorphological features is not clear. However, the framework has large capacity of the expression. The most of the terrain models seem to support the appropriateness of the framework and the modeling operator.

Accordingly, it should be noticed that the equation is an approximation and the solution does not need the strictness. It is sufficient even when the iterative method holds asymptotically.

3.2 Geomorphological requirements for the operator

The geomorphological requirements for the operator are related to various mathematical points. We here discuss only three of the most important requirements without their mathematical details.

The first is that the geomorphological law expressed by the operator does not depend on position. Geologically speaking, the law must depend on position. However, we neglect the dependence because the contour data do not have the information. Then the operator cannot have x and y as its arguments.

The second is that the operator conserves the high-frequency components of the landform. They are indispensable to the fractal feature. We can show that linear operators do not have the conservational property by considering their representation in the Fourier space. Then we must introduce a non-linear operator in order to posess the property.

The third is that the operator does not give any singuralities and strange patterns along contour lines, because a landform must be independent of the sampling pitch and the *origin of contours.* It is very difficult to satisfy the requirement because it is related to not only many mathematical points but also geomorphological points. For example, the operator must admit the fractal property of contour lines and propagate the property to the areas far from the contours. The non-linear interaction of the propragational waves from some contours must have geomorophological consistency. Hence, we can make the operator satisfy the requierment partially so far.

4 Shading examples

Here we show some shadings of the terrain models, to compare the model by the new method with the others.

We put the 50m-pitched contour data displayed in Figure 1 in computer, and obtain three terrain models by differrent methods. Note that every height of the models is exaggerated in 5 times vertically and the light is from upper-left direction.

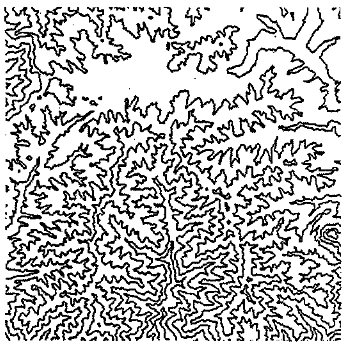

Figure 1: input contour data

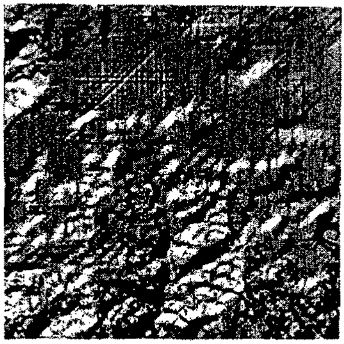

Figure 2: by average of spline curves

Figure 3: by low-pass filter

Figure 4: by the new method

In Figure 2, the shading is given by the method using the weighted average of spline curves in 4-directions. In Figure 3, the 2-dimensional low-pass filter is operated on the stacking model.

In Figure 4, the new method is used.

The landform in Figure 2 has strange patterns along the contour lines and various noises such as the starlike noise. The landform in Figure 3 is vague and smooth without any fine folds, and has the trace of contour lines on the area with low-dense contours. In Figure 4, the information of the contour data is fully used. The fine geomorphological features as well as the lines of valleys and ridges are extracted from the contours without any noises mentioned above.

5 Conclusion

We present a novel method to give a digital terrain model from contour data. The method is consituted on a boundary value problem for the non-linear operator expressing the geomorphological features.

In this result, the information of contour data is fully used in the computation and a fine realistic model with physical consistency is given. The terrain models not only become free from the unnatural noises which are unavoidable by the conventional methods, but also have the fractal feature naturally extracted from the contours.

References

[1] Yoeli. P., 1986. Computer executed production of a regular grid of height points from digital contours. The American Cartographer, Vol. 13, pp. 219-229.

[2] Christiansen, H. N. and Sederberg, T. W., 1978. Conversion of complex contour line definitions into polygonal element mosaics. Computer Graphics, Vol. 12, pp. 187-192.

[3] Ganapathy, S. and Dennehy, T. G., 1982. A new general triangulation method for planar contours. Computer Graphics, Vol. 16, pp. 69-75.

[4] Inoue. H., 1986. A least-squares smoth fitting for irregularly spaced data: finite-element approach using the cubic B-spline basis. Geophysics, Vol. 51, pp. 2051-2066.

[5] Chiba, N., Muraoka, K., Misawa, M. and Saito, N., 1987. Interpolation for digital terrain modeling. Trans. of Information Processing Society of Japan, Vol. 28, pp. 1121-1133. (in Japanese)

[6] Ikeda, T., Ueda, M. and Kunii, T., 1993. Geomorphology based on homotopy model. Journal of the Japan society of photogrammetry and remote sensing Vol. 32, pp. 18-24. (in Japanese)

[7] Nogami, M. and Sugiura, Y., 1986. The exercise on mathematical geography by personal computer. Kokonsyoin, pp. 103-135, Tokyo. (in Japanese)