Spatial Interpolation of Airborne Laser Scanning Data with Variable Data Density

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Abstract. Airborne laser scanning data are increasingly available for various applications including digital elevation modeling. One of the biggest challenges for its successful use is the varying data density caused by land cover properties. Areas with dense canopy cover have much lower data density than the open areas. This varying data density may pose a problem for some spatial interpolation methods, such as Regularized Spline with Tension. In this study, we propose a methodology to eliminate the interpolation artifacts caused by the varying data density by a proper selection of data points entering the interpolation process. Using the application example from the Slovak karst area, we demonstrated the applicability of the presented approach.

Keywords: Spatial interpolation, Airborne laser scanning, Data density

1. Introduction

The main use of the airborne laser scanning (ALS) data is in producing highly detailed digital models of surfaces sampled by the laser beam which can be reflected (returned) from several surface levels and recorded (Wehr & Lohr, 1999). Spatial density of the bare ground returns varies with the land cover properties as they determine the capacity of the laser to reach the ground through the canopy. The varying density may pose a problem for some interpolation methods frequently used to compute a grid-based digital terrain model (DTM). Numerous artifacts such as abrupt spikes or squared terraces may occur in the contact zones of high and low data density. Hofierka & Cebecauer (2007) suggested some solutions albeit not focusing on the specifics of the ALS data. In contrast to traditional data sources, the ALS data comprise a huge number of points entering the interpolation with very high computer processing power demands. There are studies pro-

posing reduction of the input data while the output surface is satisfactory (Liu 2008). However, if applied globally, points over smaller landforms can be lost. The aim of the presented research is to tackle the issue of varying data density by a selective reduction of the input data and appropriate parameterisation of the interpolation method to preserve important geomorphic features in a karst terrain (e.g. dolines).

2. Methods and Data

2.1. Spatial interpolation using regularized spline with tension

The Regularized Spline with Tension (RST) is an interpolation function belonging to the group of radial basis functions with two imposed interpolation conditions: the function should pass through the data points and, at the same time, it should be as smooth as possible. These conditions are expressed by the minimization of deviations from the measured points and its smoothness seminorm (Mitas and Mitasova 1999):

$$\sum_{j=1}^{N} \left| p_j - f\left(\mathbf{x}_j\right) \right|^2 w_j + w_0 I\left(f\right) = minimum \tag{1}$$

where $f(\mathbf{x})$ is the RST function, p_j are the values measured at discrete points $\mathbf{x}_j = \left(x_j, y_j\right)$, $j = 1,...,N$ within a region of a 2-dimensional space, w_j, w_0 are positive weighting factors and $I(f)$ is the smoothness seminorm. For , the function $f(\mathbf{x})$ passes exactly through the measured points. The general solution of the minimization problem given by equation (1) can be expressed as a sum of two components (Mitasova & Mitas, 1993):

$$f\left(\mathbf{x}\right) = T\left(\mathbf{x}\right) + \sum_{j=1}^{N} \lambda_j R\left(\mathbf{x}, \mathbf{x}_j\right) \tag{2}$$

where $T(\mathbf{x})$ is a "trend" function and $R\left(\mathbf{x}, \mathbf{x}_j\right)$ is a radial basis function with an explicit form depending on the choice of the smoothness seminorm $I(f)$. The smoothness seminorm has been designed to include several useful interpolation properties such as an explicit form, multivariate formulation, smooth derivatives of higher orders, variational freedom through tension and anisotropy.

In a bivariate (2-D) formulation, the RST function generally defined by (2) has the following explicit form (Mitasova et al., 1995):

$$f\left(\mathbf{x}\right) = a_1 + \sum_{j=1}^{N} \lambda_j \left\{ -\left[E_1\left(\rho\right) + \ln \rho + C_E \right] \right\}, \tag{3}$$

where $\rho = (\varphi r/2)^2$, $r^2 = (x - x_j)^2 + (y - y_j)^2$ is the squared distance, C_E =0.577215 is the Euler constant, $E_1(.)$ is the exponential integral function, and φ is a generalized tension parameter which provides the control over the influence of derivatives of certain order on the resulting function. The function is implemented in GRASS GIS as the *v.surf.rst* command (Neteler and Mitasova, 2008). In this command, the interpolation process is controlled by the following set of parameters:

- tension φ,

- smoothing w,

- anisotropy (θ, s), where θ, s are the rotation and scale, respectively,

- minimum and maximum distances between points.

Tension φ, smoothing w and anisotropy (θ, s) are the key internal parameters controlling the character of the resulting surface. The tension parameter controls the behavior of the resulting surface from a thin membrane to a stiff steel plate. The RST method is scale dependent and the tension works as a rescaling parameter (Neteler & Mitasova 2008). A high tension "increases the distances between the points" and reduces the range of impact of each point, low tension "decreases the distance" and the points influence each other over a longer range. The tension parameter plays a key role in areas with a steep change of modeled phenomenon where overshoots and undershoots of the interpolated surface may occur. A surface interpolated with a high tension behaves like a membrane (rubber sheet stretched over the data points) with peak or pit in each given point and everywhere else the surface goes rapidly to trend. A surface with a very low tension behaves like a stiff steel plate and overshoots can appear in areas with rapid change of gradient (Hofierka, 2005). Minimum and maximum distances between points control the number of points that are actually used in interpolation after reading the input data. However, this parameter internally influences the effect of the tension, because the tension works as a distance-scaling factor. Therefore, the tension can be set with or without normalization of data. The data density does not affect the normalized (rescaled) tension parameter.

Using the smoothing parameter *w*, the RST behaves like an approximation function, i.e. the resulting surface does not pass through the given points, but approximates the input values. This parameter is useful in modeling noisy data, where higher smoothing can filter out the noise, or alternatively, when the phenomenon needs to be modeled at a lower level of detail.

The anisotropy parameters (θ, s) can be used for interpolation of anisotropic data. The orientation of the perpendicular axes characterizing the anisotropy is defined by the rotation parameter θ and the scaling ratio of the perpendicular axes (a ratio of axes sizes) is defined by the scale parameter s. These parameters scale distances (i.e., the value of tension) in 2 perpendicular directions that should fit the spatial pattern of the anisotropic phenomenon.

The number of points used for interpolation by the v.surf.rst module in GRASS GIS is controlled by 4 parameters: dmin – minimum distance between points (to remove almost identical points), dmax – maximum distance between points on isoline (to insert additional points), segmax defining the maximum number points in the interpolation segment and npmin – minimum number of points used for interpolation in a segment (Neteler & Mitasova, 2008). The segmentation procedure of the module divides the whole area into a set of overlapping segments to ensure a smooth connection of the segments to the final surface.

These parameters can be selected empirically, based on the knowledge of the modeled phenomenon, or automatically, by minimization of the predictive error estimated, for example, by a cross-validation procedure (Hofierka, 2005, Hofierka et al., 2007).

Evaluating the interpolation quality

Evaluation of the interpolation accuracy requires comparison of the actual (measured) data values with the interpolated (estimated) values. Such a comparison leads to errors (or residuals) at the given points (Weng, 2006). To quantify the residuals various statistical measures are used, such as the mean absolute error (MAE), the mean error (ME), and the root mean squared error (RMSE). These error measures are defined as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|z_i^* - z_i\right|, \tag{4}$$

$$ME = \frac{1}{n}\sum_{i=1}^{n}\left(z_i^* - z_i\right), \tag{5}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(z_i^* - z_i)^2}, \tag{6}$$

where z^*_i is the interpolated (estimated) value at location i and z_i is the actual (measured) value.

In case of exact interpolators the residuals between the interpolated and measured values are rather low because this is an inherent feature of these methods. Still, in case of very rapid changes in terrain this comparison provides valuable information about the quality of interpolation. However, this comparison says very little about the quality of interpolation in areas between the measured points. To minimize the drawbacks of this direct comparison and provide an assessment of the predictive capabilities of the interpolation method in area without data, a specific evaluation method called cross-validation (CV) or jack-knife is widely used. This method is based on removing one input data point at a time, performing the interpolation for the location of the removed point using the remaining samples and calculating the residual between the actual value of the removed data point and its estimate. The procedure is repeated until every sample has been, in turn, removed. This form of CV is also known as the ''leave-one-out'' method (Hofierka et al., 2002).

The CV procedure is especially suitable for relatively dense data sets, since removing points from already under-sampled areas can lead to misrepresentation of the surface to be interpolated (the surface is smoothed). The minimum statistical errors calculated by CV can be used to find the optimum interpolation parameters (Mitasova et al., 1995), (Hofierka et al., 2002). However, Hutchinson (1998) has found that CV does not always represent a reliable estimate of the model error, especially when a short-range correlation in data is present. Hofierka (2005) has shown the limited applicability of CV to the optimization of RST parameters in areas where input data are not sufficiently sampled especially in anomaly and local fluctuation areas with a higher need for representative sampling. Also, it has been shown that the RST parameters are flexible enough to produce interpolation results that reflect the behavior of the modeled phenomenon even for less dense data sets. Another serious drawback of the CV method is its high computational demand in case of large datasets. Interpolation must be iteratively repeated for every removed point and every combination of interpolation parameters which is also the case of ALS data.

Hofierka et al. (2007) have suggested that the evaluation of interpolation accuracy can be also assessed using an evaluation dataset containing data not used in interpolation. For each evaluation point the error between actual and interpolated value is calculated and the overall accuracy is tested. This evaluation dataset can be taken from the independent measurement of points or the original dataset using points selected by a random generator. The selected points are then removed from the interpolation dataset.

2.2. Study area

The ALS data used in this research represent a 2 by 2 km portion of the Slovak Karst, East Slovakia. The area is mostly wooded with occasional meadows and scrubs comprising a plateau dissected by a deep canyon and a few occasional dolines (Figure 1). The altitude ranges between 540 to 704 m a.s.l. The data were acquired in the leaf-on conditions in 2009 and supplied as a filtered point cloud. The supplier claims the vertical root mean square error (RMSE) of 23 cm. The sample area contains 217 984 points classified as bare ground of spacing varying between 1–80 m with the average data density around 0.054 point/m^2 (Figure 2). Spatial distribution of points is very uneven with a higher data density associated with open (grass) areas with the average data density around 0.16 point/m^2 and the lowest data density in compact forest areas with dense canopy cover (0.03 point/m^2).

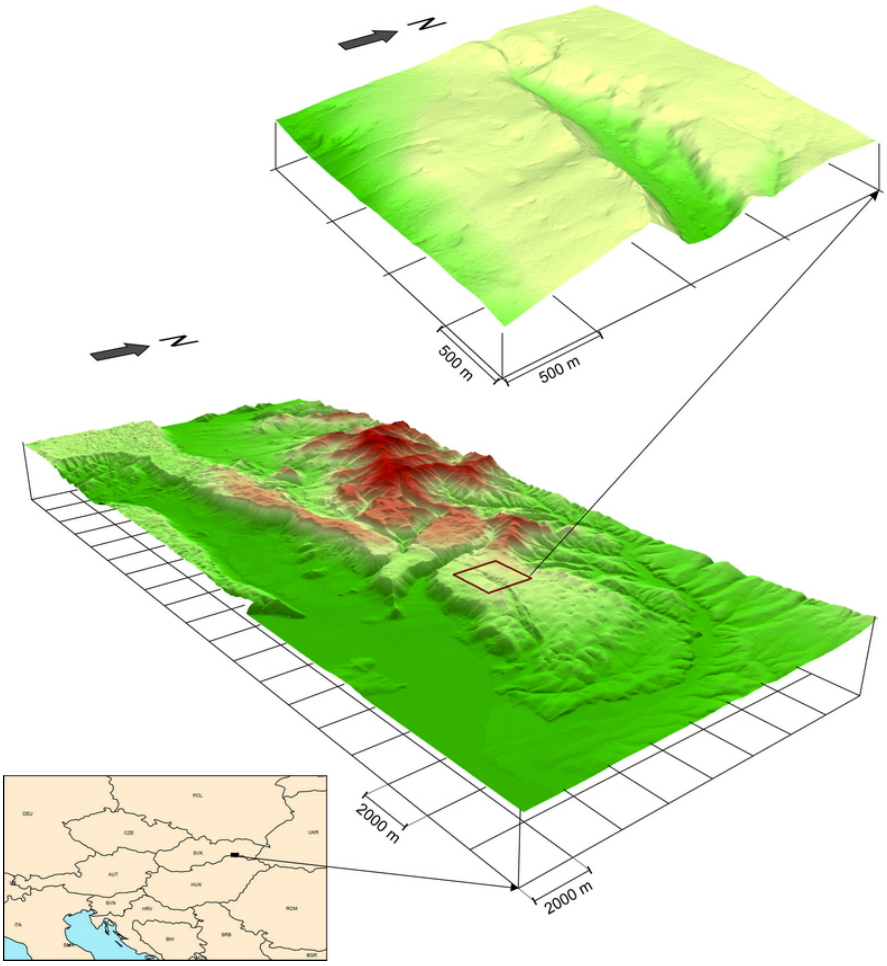

Figure 1. Position of the study area.

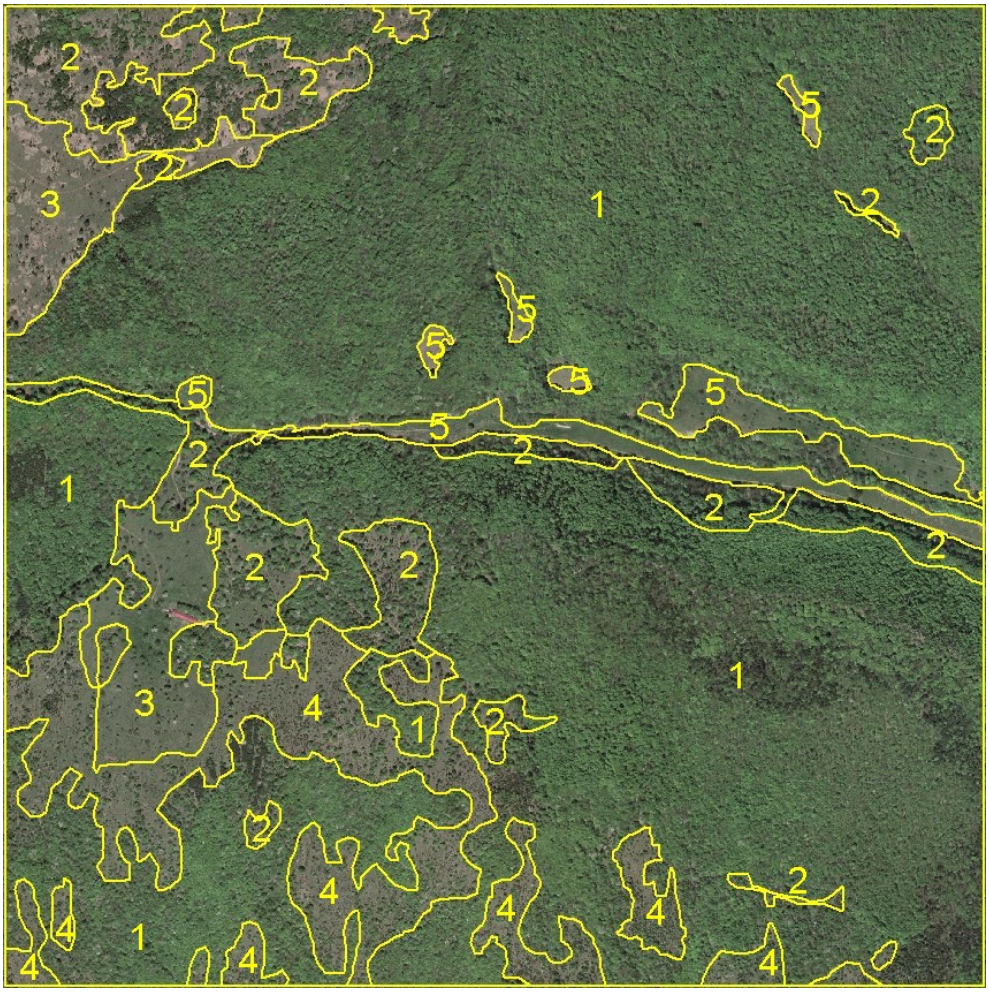

Figure 2. Orthophotomap of the study area with land cover classes (1 - forest with dense canopy cover, 2 - forest with sparse canopy cover, 3 - grass with sparse trees, 4 - grass and shrub, 5 – grass).

3. Results and Discussion

The v.surf.rst interpolation module implemented in GRASS GIS version 6.4.2 has been used to compute the DTM of the study area. The spatial resolution of the grid-based DTM was set to 2 m which should be sufficient to capture most of the spatial variation in elevation including specific karst relief features, dolines, with the mean size in diameter of a few tens of meters.

The initial DTM was computed using the tension set to 20 and the rest of parameters set to default values (Table 1). The resulting surface showed a few interpolation artifacts such as visible segments caused by the uneven data density and segmentation procedure implemented in the module. The solution recommended by the GRASS manual page is to increase the number of points taken for the segment computation with larger overlapping areas around the segments using the npmin parameter. The npmin parameter was set to 400 and dmin to 2 meters (Table 1). However, increasing the number of points using the npmin parameter has a limited usability for larger datasets because this substantially increases the computational time (approximately with a power of three). Despite it helped to improve the quality of interpolation, some interpolation artifacts in the resulting surface were still present.

Hofierka & Cebecauer (2007) have suggested that the interpolation artifacts caused by the segmentation procedure can be also minimized by a more even distribution of data points. They used contour data from topographic maps and photogrammetric imagery. The initial DTM with interpolation artifacts was sampled to include data points representing the surface in areas with no input data points. While this approach may lead to satisfactory results also for the ALS data, the problem of efficient computation of large datasets consisting of millions of data points is not still fully addressed.

Our analysis showed that the ALS data density is usually very high on areas with no canopy cover (Figure 2 vs. Figure 3a). Depending on the purpose of representing specific relief features we can safely reduce the number of data points taken by the v.surf.rst module using the dmin parameter even more than the limit of 2 meters (for example, with dmin=6 meters) without losing too much information and still preserving the necessary accuracy of the resulting surface needed to represent specific karst landforms. However, to minimize the possible loss of information in areas with sparse data we have used a selective reduction of data points with the chosen limit of 6 meters. The selection of points is based on the interpolation error registered for each point during interpolation with the dmin parameter set to 2 meters. All points with an interpolation error lower than 0.25 m were removed and only points with a higher error were kept during the computation with the dmin parameter set to 6 meters (Table 1). This assumes that the data is sufficiently reliable and accurate and the interpolation error is caused mainly by abrupt changes in the resulting surface. To preserve the character of the interpolated surface and compare the interpolation results, we used the unnormalized tension parameter of the v.surf.rst module (-t flag) set to 185 (which correspondents to a rescaled tension of 20). The main benefit of the suggested approach is a reduction of interpolation artifacts arising from the

uneven spatial distribution of the input points and much higher speed of computation what is so important for massive datasets produced by the laser scanning.

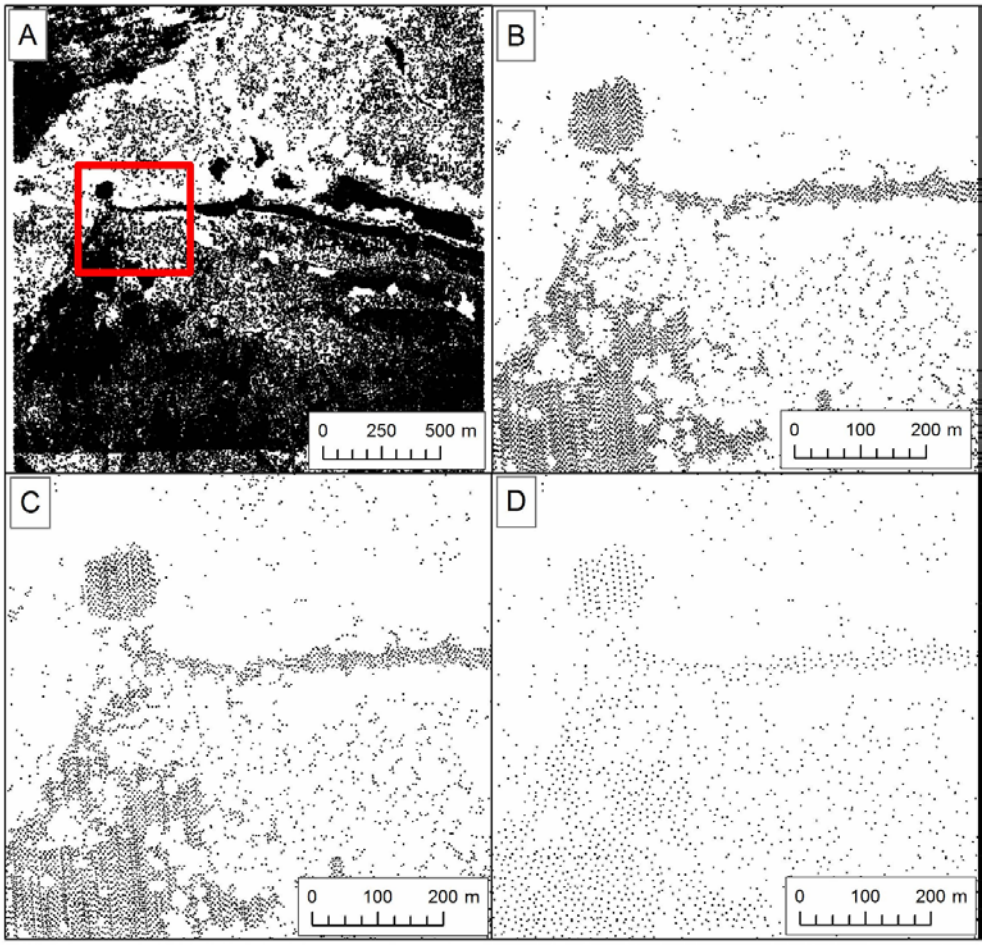

Figure 3. Input ALS points. A – entire study area. B – detail of the area in the red square on A. C – reduced dataset with a minimal spacing of input data set to 2 m, D – reduced dataset with a minimal spacing of input data set to 6 m.

The comparison of interpolation accuracies achieved by the suggested set of parameters is presented in Table 1 and Table 2. While Table 1 presents the interpolation accuracy at given points, Table 2 presents the predictive error of the interpolation method with various parameterisation settings using an evaluation set of 1000 randomly selected points withheld from further interpolation. The comparison shows clearly the drawbacks of evaluation of interpolation accuracies at given points. The best results by RMSE were achieved for the setting with dmin=6 meters (RMSE=0.1432 m). However,

Table 2 shows that the best results were achieved for dmin*=6 meters with additional points with the highest interpolation errors identified during the computation with dmin set to 2 meters. In contrast, the best result identified in Table 1 is the worst in Table 2.

The overall interpolation results of the RST method and v.surf.rst module are very good because in all cases the RMSE is very close to the declared overall accuracy of the ALS data (RMSE = 0.23 m). However, the interpolation artifacts were clearly visible in the parameterisation using npmin=300. Almost complete elimination of interpolation artifacts can be seen using npmin=400 and further reduction of point using dmin=6 meters while still preserving the mapped geomorphic features (Figure 4).

The lowest RMSE was achieved using the suggested approach of selective data points reduction (RMSE=0.2195 m). Another benefit of the proposed method is also in the substantial increase in the speed of computation (Table 1).

	NP	RE	ME	MAE	RMSE	CT
tension=20, dmin=1.0, npmin=300	183 418	17.23	-0.000028	0.1068	0.1596	65
tension=20, dmin=2.0, npmin=400	136 502	16.05	-0.000022	0.1191	0.1797	120
tension*=185, dmin=6.0, npmin=400	44 682	7.29	0.000011	0.0959	0.1432	23
tension*=185, dmin*=6.0, npmin=400	62 640	13.98	0.000015	0.1456	0.2196	44

Table 1. Interpolation accuracy at given points (tension* - um-normalized tension, dmin* - selective dmin, NP - number of points, RE - range of errors, ME - mean error, MAE - mean absolute error, RMSE - root mean square error, CT - computational time in minutes).

	ME	MAE	RMSE
tension*=185, dmin=2.0, npmin=400	-0.02125	0.1676	0.2447
tension*=185, dmin=6.0, npmin=400	-0.02163	0.1782	0.2592
tension*=185, dmin*=6.0, npmin=400	-0.01137	0.1570	0.2195

Table 2. Interpolation accuracy using an evaluation dataset of 1000 randomly selected points.

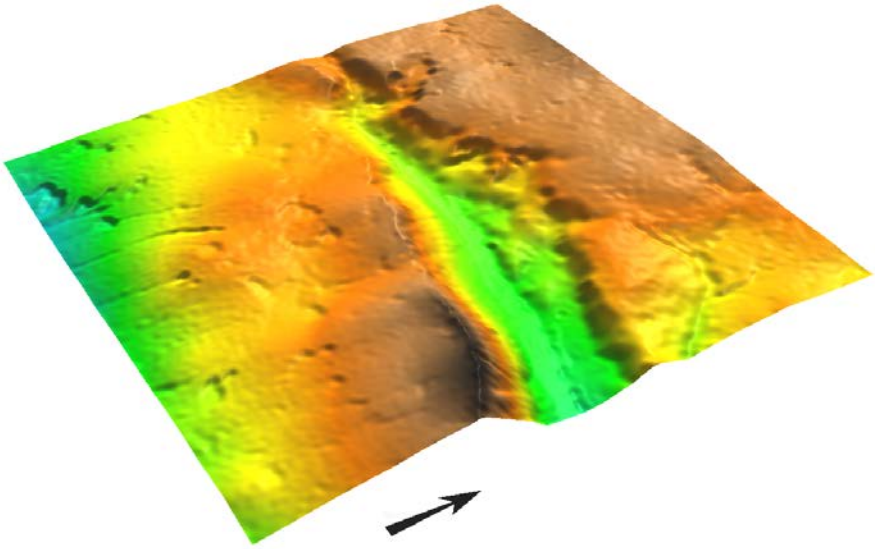

Figure 4. Resulting DTM. Specific karst geomorphic features remained preserved in the interpolated surface.

4. Conclusion

This paper explores the methodology of efficient processing of ALS elevation data with a variable spatial density using the RST interpolation method. We demonstrated that the interpolation artifacts can be minimized by controlling the number of data points used in the interpolation process based on the minimal distances between the points as well as the selection of the most important points derived from the analysis of interpolation errors.

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