

# Extending Adaptive Composite Map Projections with Wagner's Transformation Method

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**Abstract.** Adaptive composite map projections transform projections as the user changes the scale of the web map. The equal-area version of the original adaptive map projection composite uses the Lambert azimuthal equal-area projection for regional maps, and three equal-area world projections, namely the Hammer, the Eckert-Greifendorff, and the quartic authalic projections. These three projections show poles as points. With Wagner's transformation method, additional projections can be added to the adaptive composite map projections. These additional projections can be transformed to the Lambert azimuthal projection; they are equal-area and they show poles as curved or straight lines.

**Keywords:** Adaptive composite map projections, multi-scale map, Wagner's transformation method, web mapping, web Mercator

## 1. Introduction

The majority of web maps today use the Web Mercator projection. While this projection has advantages for maps at large scales, it is not well suited for small-scale mapping because it shows areas close to poles with enormous areal distortion. There are many alternative projections available for small-scale world maps that can be used instead of the Web Mercator projection. Hence, for an interactive scalable map, different projections should be used at different map scales. This approach requires a smooth transformation between projections that preserves distortion characteristics. A simple approach would consist of blending projections using interpolation with a weighted mean, but this blending would not retain distortion characteristics. For example, when linearly interpolating two equal-area projections, the resulting projection is generally not equal-area. Moreover, interpolation could result in curvy and folded graticules.

## 2. Adaptive Composite Map Projections

Recently, the adaptive composite map projections (Jenny 2012) have been presented as an alternative to solve these issues. In this composite of map projections, different projections are selected according to the scale of the map and the location of the mapped area. With the proper choice and combination of projections, web maps can be equal-area for any map scale. However, the number of possible equal-area projections for very small scales is currently limited to the Hammer, the Eckert-Greifendorff, and Siemon's quartic authalic projections. These three projections can be converted into Lambert's azimuthal projection, the medium-scale projection used in the adaptive composite map projection. The transformation between the small-scale and medium-scale projection is made by multiplying the abscissa of Lambert's azimuthal projection with a selected factor (marked as  $B$  on the Figure 1) while at the same time dividing longitudes with the same factor value. David Aitoff introduced this technique in 1889 (Snyder 1993). The value of the factor  $B$  in the adaptive composite map projection changes with map scale (Jenny 2012). Figure 1 presents a sequence of map projections where the value of the factor  $B$  grows from 1 to infinity.

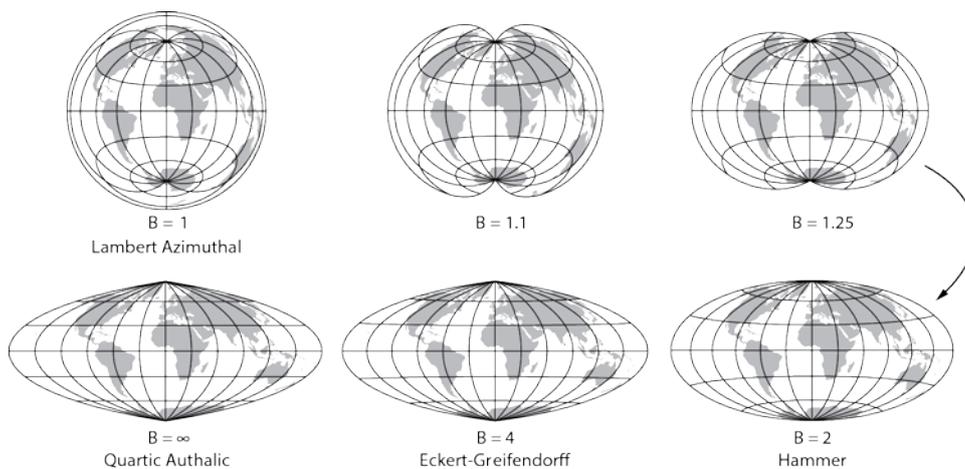


Figure 1: Transforming Lambert's azimuthal projection by changing the factor  $B$ .

## 3. Wagner's Transformation Method – “Umbeziffern”

In 1932, the German cartographer Karl Heinrich Wagner suggested a transformation technique for the development of new map projections also known by its German name “Umbeziffern,” meaning renumbering (Canters 2002). This method first maps the longitude and latitude values using two parameters onto a smaller segment of the globe, and then projects the geo-

graphic coordinates using an existing projection. After projection, a part of the graticule is enlarged to the parent projection's scale with a parameter that adjusts the graticule to the preferred equator and central meridian ratio (Wagner 1982). The result is a new map projection. Wagner (1949) presented three methods with one maintaining the area distortion characteristics of the parent projection. With this method, different equal-area projections with straight or curved parallels can be created. For example, Wagner (1949) derived the Wagner VII projection from Lambert's azimuthal equal-area projection using this method. Karl Siemon (1937) proved that Aitoff's transformation method is a special case of Wagner's transformation. Aitoff's method results in projections where poles are represented as points. With Wagner's transformation method, projections representing poles as lines can be created.

#### **4. Extending Adaptive Composite Map Projections with Wagner's Transformation Method**

Wagner's transformation method is a powerful mechanism that can be applied to extend adaptive composite map projections. It enables smooth and equal-area transformation between two related projections by changing the values of one ratio parameter and two parameters that define the range of the longitude and latitude. In the context of adaptive composite map projections, these parameters change with map scale.

Extending adaptive composite map projections with Wagner's transformation method makes it possible to enlarge the set of small-scale map projections. Three different groups of equal-area world map projections can be added to adaptive composite map projections. The first group includes projections with curved parallels and pole lines, like the Wagner VII projection. The second group consists of true pseudocylindrical projections that represent pole lines and parallels as straight lines. The third group of projections includes equal-area cylindrical projections with any standard parallel.

#### **5. Conclusion**

Wagner's transformation method extends the set of equal-area small-scale map projections for adaptive composite map projections. While creating an interactive scalable map, the cartographer may now use world map projections that represent poles as lines or even use cylindrical projections with a rectangular shape — although their usage cannot generally be recommended for world maps (Robinson 1990).

## References

- Canters F (2002) *Small-Scale Map Projection Design*. CRC Press, London
- Jenny B (2012) Adaptive composite map projections. *IEEE Transactions on Visualization and Computer Graphics (Proceedings Scientific Visualization / Information Visualization 2012)* 18(12): 2575–2582
- Robinson AH (1990) Rectangular world maps —no!. *The Professional Geographer* 42(1): 101–104
- Siemon K (1937) Flächenproportionales Umgraden von Kartenentwürfen. *Mitteilungen des Reichsamts für Landesaufnahme* 13(2): 88–102
- Snyder JP (1993) *Flattening the Earth: Two Thousand Years of Map Projections*. University Of Chicago Press, Chicago
- Wagner KH (1949) *Kartographische Netzentwürfe*. Bibliographisches Institut, Leipzig
- Wagner KH (1982) Bemerkungen zum Umbeziffern von Kartennetzen. *Kartographische Nachrichten* 32: 211–18