A KIND OF ADJUSTABLE MAP PROJECTION WITH " MAGNIFYING GLASS" EFFECT

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Abstract

This paper proposes a new kind of varioscale map projection, which has the effect of "magnifying glass" that can show the signficant regions with larger scale while the scale of surrounding parts remains constant. The range form and variation of the magnified region can be designed and adjusted freely. The proposed method can basically include the existing varioscale projection methods and overcome their limitations as well as produce more optimal and wider results.

1 Introduction

The varioscale map projection, which can show the signficant regions and contents with larger scale by means of mathematics and projection distortion, is a new kind of practical projection. As a result of using it, the map information capacity can be increased and the map function can be improved. So far some scholars have proposed several methods for varioscale map projection. These methods are valuable but not mature, their main defect is that the scale variation can't be controlled effectively, Generally speaking, an ideal varioscale map projection should possess the ability to control the scale variation (such as the variation of form, magnitude and range) and the distortion distribution given by scale variation is flexible and convenient, so that the scale can be variable in interested regions and the distortion brought by scale variation can be avoided in other regions. The aim of this paper is find a new kind of varioscale map projection with these functions.

2 Principle

In general the varioscale map projection can be considered as a coordinate transformation from the plane of a original map to the plane of varioscale map. In view of this, let us assume that $P(x,y)$ is a point on the plane of original map, $P'(x',y')$ is the corresponding point of $P(x,y)$ on the plane of varioscale map, $O(x_0,y_0)$ is the focus, θ is the azimuthal angle of coordinate and R and r are the distances from point P and point P′ to point O respectively. In order to control the scale variation, we can design the form of scale variation beforehand. Naturally, it should be a monodrome and continuous function of $R: f(R)$. According to the meaning of varioscale, we have

$$r = Rf(R) \quad (1)$$

By preceeding assuming, the following equations can be obtained easily

$$\begin{cases} x - x_0 = R\cos\theta \\ y - y_0 = R\sin\theta \end{cases} \quad (2)$$

$$\begin{cases} x' - x_0 = r\cos\theta \\ y' - y_0 = r\sin\theta \end{cases} \quad (3)$$

Substituting the expression (1) into equations (3), we now have

$$\begin{cases} x' - x_0 = f(R)R\cos\theta \\ y' - y_0 = f(R)R\sin\theta \end{cases} \quad (4)$$

Again, substituting the equations (2) into (4), we have consequently

$$\begin{cases} x' = x_0 + f(R)(x - x_0) \\ y' = y_0 + f(R)(y - y_0) \end{cases} \quad (5)$$

Where R is a function of x and y. It discribes a circular region, but in fact, it also discribes other shaped regions extensively. We might as well let $R = \varphi(x,y)$, then the equations (5) can be written as

$$\begin{cases} x' = x_0 + f(\varphi(x,y))(x - x_0) \\ y' = y_0 + f(\varphi(x,y))(y - y_0) \end{cases} \quad (6)$$

In order to control the range of scale variation, we may rewrite the function $f(\varphi(x,y))$ into a general form:

$$f(\varphi(x,y)) = \begin{cases} f_1(\varphi(x,y)) & 0 < \varphi(x,y) \leqslant a_0 \\ f_1(\varphi(x_1,y_1)) & \varphi(x,y) > a_0 \end{cases} \quad (7)$$

Where f_1 means the scale variation form as f, and a_0 means the critical value, which is a parameter to control the range of scale variation, $\varphi(x_1,y_1) = a_0$.

It is equations (6) and (7) that describe a kind of adjustable map projection with "magnifying glass" effect. In "magnifying glass"(i. e. $0 < \varphi(x,y) \leqslant a$), the scale varies as the form of function $f_1(x,y)$, and outside of the "magnifying glass"(i. e. $\varphi(x,y) > a_0$), the scale remains constant $f_1(\varphi(x_1,y_1))$. Furthermore, the "maginfying glass" is adjustable. For example, the magnifying form and variation can be adjusted by designing function f_1; the shape of "magnifying glass" can be adjusted by designing function φ and parameter a_0 respectively.

3 Designing the magnifying form of "magnifying glass"

When we apply the proposed formulas (5)—(7) in practice, the function f, which indicates the magn-

ifying form of "magnifying glass" or scale variation, should be set up first. We *now give some useful* designs for function f by the analysis method. In the following discussion, we might as well fix the function φ in formulas (5)—(7): $\varphi(x,y)=R=\sqrt{(x-x_0)^2+(y-y_2)^2}$. In addition, we take the grids of cylindrical equidistant projection as a original map(see fig. 1). All figures shown in later parts are drawn by the method of computer graphics and the computational programs are omitted.

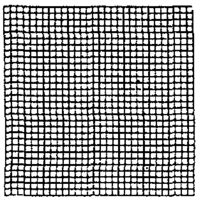


fig. 1

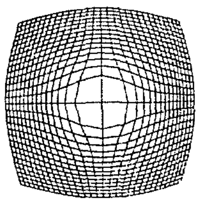


fig. 2

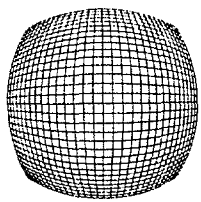


fig. 3

When the scale variation on a small portion around the focus is required to vary quickly, we can let $f(R)=\alpha(1+R)^{\beta}$ in equations (6).

Where α and β are adjustable parameters that can control the amplitude of scale variaton (in following, we will continue to use them, i. e. the meaning of α and β will be remained). The projection effect can be seen in fig. 2.

When the scale variation on a larger portion around the focus is required to vary slowly. We can let $f(R)=\alpha e^{-\beta R}$ *in equations*(6). *The projection effect can be seen in fig.* 3.

4 Designing the shape of "magnifying glass"

In order to limit the scale variation within the considered regions and improve the effect of variocale map projecton we need to control the shape of "maynifying glass". In other words, to set up the function φ *in* equations (6) and (7).

- Circular "magnifying glass"

In practice, circular "magnifying glass" is used frequently. As long as the function f_1 in formula (7) had to be designed or chosen concretely, and $\varphi(x, y)$ be given by $R=\varphi(x,y)=\sqrt{(x-x_0)^2+(y-y_0)^2}$, we can get corresponding circular "magnifying glass". For example, taking $f_1=\alpha\sqrt{1-(R/\beta)^2}$. We can get the varioscale map projections with circular "magnifying glass" effect shown as fig. 4.

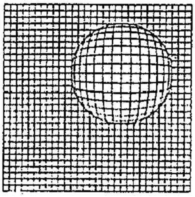


fig. 4

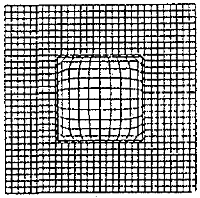


fig. 5

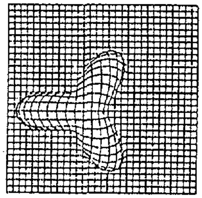


fig. 6

• Non-circular "magnifying glass"

Besides circular shape, common shapes of "magnifying glass" are elliptical, rhombic (including square), trilobate (including triangular) and quartlobate. Once function f_1 in formula (7) be set (where we might as well choose $f_1=\alpha\sqrt{1-(R/\beta)^2}$), and function $\varphi(x,y)$ be given as following forms:

$\varphi_1(x,y)=A|x-x_0|+B|y-y_0|$

$$\varphi_2(x,y)=\frac{3}{2}\sqrt{\left[\left(x-\frac{A(X^2-Y^2)}{X^2+Y^2}\right)\right]^2+\left[Y-\frac{2AXY}{X^2-Y^2}\right]^2}$$

Where A and B are adjustable parameters that can control the shape variation, $X=x-x_0$, $Y=y-y_0$. Then we can get corresponding "magnifying glasses" with designed shapes (see figures 5 and 6). Since the discussed shapes and their directions can be adjusted by not only function φ but also the parameters arbitrarily, these non-circular "maynifying glass" can already meet the needs for shape variation of "magnifying glass".

5 Polyfocal projection

In order to obtain polyfocal projection with "magnifying glass" effect, we rewrite the formula (6) as

$$\begin{cases} x' = x + (f(\varphi(x,y)) - 1)(x - x_0) \\ y' = y + (f(\varphi(x,y)) - 1)(y - y_0) \end{cases} \tag{9}$$

Let $F(\varphi(x,y))=f(\varphi(x,y))-1$, equations (9) can be written as

$$\begin{cases} x' = x + F(\varphi(x,y))(x - x_0) \\ y' = y + F(\varphi(x,y))(y - y_0) \end{cases} \tag{10}$$

It is obvious that $F(\varphi(x,y))$ in (10) still means scale variation form. Equations (10) indicate that the coordinate transformation of the proposed projection with one focus is led to by $F(\varphi(x,y))$ and its "function" on the coresponding coordinate increments ($\Delta x=x-x_0$, $\Delta y=y-y_0$). From this, we can infer the polyfocal projection formula.

suppose that $O_i(x_i,y_i)$ are focuses, $P(x,y)$ is any point on the varioscale map. $R_i=\varphi_i(x,y)$ are the distances from $P(x,y)$ to $O_i(x_i,y_i)$ and $F_i(\varphi_i(x,y))$ are corresponding scale variation forms of $O_i(x_i,y_i)$ $(i=1,2,\cdots,n)$. In view of preceeding analysis for equations (10), we can obtain the following polyfocal projection formula by superposing the "functions" $F_i(\varphi_i(x,y))$ about the corresponding coordinate increments $(\Delta x_i = x - x_i, \Delta y_i = y - y_i \quad i=1,2,\cdots,n)$:

$$\begin{cases} x' = x + \sum_{i=1}^{n} F_i(\varphi(x,y))(x - x_i) \\ y' = y + \sum_{i=1}^{n} F_i(\varphi_i(x,y))(y - y_i) \end{cases} \quad f_i(\varphi_i(x,y)) = \begin{cases} G_i(\varphi(x,y)) & 0 < \varphi_i(x,y) \leqslant a_i \\ G_i(\varphi_i(x_i,y_i)) & \varphi_i(x,y) > \alpha_i \end{cases} \tag{11}$$

Where G_i mean the scale variation form as F_i, α_i mean the critical value about $O_i(x_i,y_i)$, $\varphi_i(x_i,y_i) = a_i, i=1,\cdots,n$.

It is easy to know that when F_i, φ_i are set on a similar plan of proposed projection with one focus, and n, a_i $(n=1,2,\cdots,n)$ are fixed in formula (11), we can get concrete polyfocal projection with "maynifying glass" effect. Some practical examples can be seen in figures 7, 8, and 9.

These examples indicate that the proposed polyfocal projection can produce quite abundant results owing to F_i, φ_i in formula (11) can be collocated or composed in different ways. In addition, the mutual distortion affects, which are brought about by partial distortion about each focus, can be removed successfully. Thus the thorny problem that is difficult to handle with by other varioscale map projection methods is finally solved.

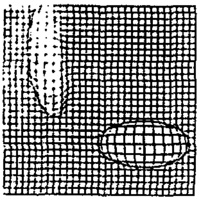


fig. 7

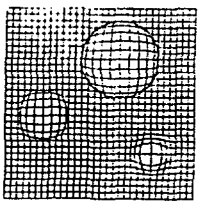


fig. 8

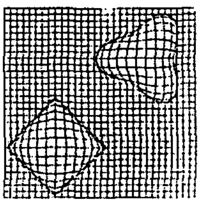
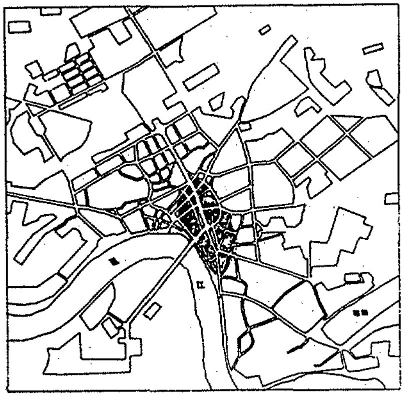


fig. 9

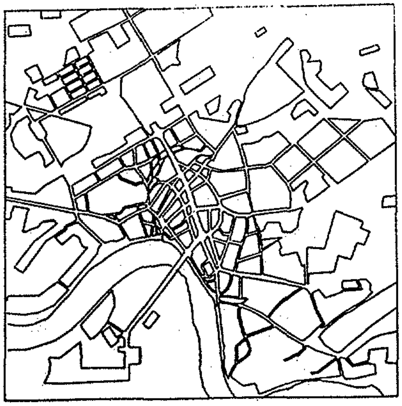
6 Practical example

Finally a traffic and tourist maps are taken as examples to illustrate the application of the proposed varioscale map projection under the environment of computer-assisted cartography.

The original map(see fig. 10(a)) is the traffic and tourist map of Nanning city in China. The dense and busy region of this city is located in the elliptic area shown as the shadowgraph of the map. If we are interested in this area, we can place it under a elliptic "magnifying glass" and get the corresponding variscale map (see fig. 10(b)), this results in that the interested area is enlarged and the contents in "magnifying glass" are represented in more detail, and at the same time, the graphs outside the "magnifying glass", which we have no interest with, remain statioinary.



(a)



(b)

Figure. 10: The example of varioscale map projection with elliptic "magnifying glass" effect

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