THE ROLE OF CARTOGRAPHY IN DESIGNING A GLOBAL SAMPLING FRAMEWORK FOR ENVIRONMENTAL MONITORING

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Abstract

Environmental monitoring and assessment is becoming a global activity as our concern with and understanding of global scale environmental issues increases. Additionally, international economic agreements such as GATT and NAFTA now include environmental compliance regulations. Scientists at the U.S. Environmental Protection Agency (EPA) and similar organizations in other countries are now asking cartographers and GIS experts if there exist global geographical data structures that will allow optimal implementation of survey sample designs, modeling of environmental processes, analyses of sampling and related data, and display of results.

In this paper we examine the role that cartographers in the United States have played in the development of global grid sampling frameworks for environmental monitoring and analysis, an EPA sponsored activity. First and foremost, cartographers have played a lead role in devising ways to define a set of regularly arranged sampling points covering the entire earth. The problem is to approximate as closely as possible a regularly spaced sampling grid. This is because not more than 12 points, the vertices of an icosahedron projected onto a sphere, can be placed on the earth equidistantly. One approach has been to model the earth as a polyhedral globe, with each face of the polyhedron being a map projection surface that optimizes the generation of a large set of approximately regularly spaced sampling points. Equal area and other projections have been developed for the faces of the octahedron and icosahedron.

Cartographers and GIS experts have also assisted in devising ways of comparing different approaches to sampling grid generation. For example, the map projection approach can be compared to the sampling grid obtained by recursively quartering the spherical triangles formed by projecting the vertices of a regular polyhedron, such as the icosahedron, onto the earth. A set of evaluation criteria, such as equality in surface area and maximal compactness of triangles formed by adjacent sampling points, has been proposed. Quantitative measures for each criterion have been defined, measures applicable to both the map projection surface and the spherical earth. We have applied these evaluation criteria to compare several grid generation approaches, and have ranked the utility of each approach accordingly.

1 Introduction

Monitoring the health of our environment and studying the physical and cultural processes driving environmental change are ever growing activities of global concern. Both global monitoring and scientific modelling rely on accurate, spatially complete databases of relevant environmental phenomena. Recent technological advances have made global data collection a reality. This has spurred recent interest in compiling and structuring global spatial data sets into both vector and raster databases, the Digital Chart of the World and AVHRR-based NDVI data being but two of the various globally complete data sets released recently.

Many global monitoring and modelling efforts will also be based on survey sampling conducted in a manner allowing statistical estimates of the properties of environmental phenomena. Survey sampling has been carried out by agencies with missions ranging from the international to local level, with each survey utilizing a sampling frame that is often a point grid. Sampling the entire earth is now scientifically feasible for certain features, which focuses attention on the need for a multipurpose global sampling grid.

For both sampling purposes and building global databases for modelling and analysis, we need a better conceptual understanding of how to partition the earth's surface. To our knowledge, no comprehensive comparison of alternative methods to partition the globe has been made.

In this paper, we go beyond our previous effort to devise a sampling grid for the entire fifty United States (White, Kimerling and Overton 1992) with the possibility of extension to the entire earth. Our objective here is to investigate more thoroughly the challenging problem of creating a partition of the earth's surface meeting the needs of global sampling and analysis. To do this, we will first examine the survey sampling requirements for a global grid. These requirements define a set of desirable geometric characteristics and associated numerical measures that serve as evaluation criteria for different partitioning approaches. We next examine how closely we can satisfy the evaluation criteria when we partition the globe into more than the twenty equilateral triangles of the spherical icosahedron. We have used both graphical and statistical analysis of area and shape measures for each recursion of the triangular partition to compare the suitablity of global grids produced by different partitioning approaches. Only area distortion will be examined in this paper.

2 Geometrical And Topological Evaluation Criteria

The statistical sampling requirements for global partitioning, particularly the need for hierarchically nested triangles equal in area that are compact with minimal shape distortion, are desirable geometrical and topological properties of the polygons into which the globe is divided. These properties and corresponding mathematical measures for each serve as evaluation criteria for alternative global partitioning schemes. It is also necessary to have a set of ideal geometric and topologic properties to serve as comparison standards. One such set would be the properties of the regular platonic solids (tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron) projected onto the spherical globe. The twenty identical spherical triangular faces of the icosahedron, for example, exhibit the following properties:

- the twenty triangles completely cover a sphere, without gaps or overlaps
- a point on the sphere will be in only one triangle, except if positioned exactly on a triangle edge or vertex
- each triangle is identical in surface area (1/20th the total surface area of the globe)
- each triangle is equilateral in shape (edge lengths and interior angles are identical)
- the centroids of the twenty spherical triangles form a regular triangular point grid such that neighboring points can be connected by geodesic lines (great circle segments) to form regular spherical pentagons.

We can now imagine systematically subdividing each of the twenty triangles of the icosahedron. The number of sub-triangles formed in the initial partitioning of the icosahedron face by any of the above methods is termed the frequency of subdivision. A 2-frequency subdivision by edge bisection produces four sub-triangles, and is also called a "four-fold" partitioning of the icosahedron face. Similarly, a 3-frequency subdivision created by dividing edges into thirds produces a "nine-fold" partitioning. Sub-triangles determined from higher frequencies of subdivision may be required for some applications, but these points can also be obtained in a hierarchically nested manner by recursively subdividing each edge at the same frequency of subdivision. If we could somehow do this partitioning in a "perfect" geometrical manner, the sub-triangles at each recursion level would have the following properties:

- sub-triangles completely covering the triangle at the next higher level in a nested hierarchical manner
- all sub-triangles equal in surface area ($1/n^2$ of the surface area of the triangle at the next higher level of recursion, where n is the frequency of subdivision)
- all subtriangles equilateral in shape with equal length edges and equal interior angles
- sub-triangle centroid points form a regular triangular grid and neighboring points can be connected to form regular hexagons on the sphere, with the exception of regular pentagons formed by centroid points adjacent to each of the twelve icosahedron vertices.

For over two thousand years we have known that this set of properties is an ideal, impossible to achieve for any partitioning of the sphere finer than the twenty triangles of the icosahedron. However, we may be able to maintain the first two properties, or maintain the first and approximate the latter three with different global partitioning methods. These properties of the icosahedron and the other platonic polyhedra have been organized by Goodchild (1994) into the list of general evaluation criteria for global partitioning found in Table 1.

1. The domain is the globe (sphere or spheroid)
2. The area partition exhaustively covers the domain
3. Partitions are equal in surface area
4. Partition areas are compact
5. Partition areas are equal in shape
6. Partition areas have the same number of edges
7. Edges of partition areas are of equal length
8. Edges of partition areas are straight on some map projection
9. Partition areas form a hierarchy preserving some (undefined) properties for $m<n$ areas
10. Each grid sample point in the domain is contained in one partition area
11. Grid sample points are maximally central within partitions
12. Grid sample points are equidistant
13. Grid sample points form a hierarchy preserving some (undefined) properties for $m<n$ points
14. Addresses of grid sample points and partition areas are regular and reflect other properties.

Table 1: Criteria for evaluation of global partitioning models
for environmental monitoring and analysis

3 Geometric And Topologic Measures For Evaluation Criteria

Mathematical measures or topological checks for each evaluation criterion in Table 1 are required when comparing partitioning methods. These may be applied singly, or in combination. A few of the many possible surface area measures, shape indices, and completeness checks are presented below, using the icosahedron as an example.

3.1 Surface Area Measures

The surface area of each sub-triangle at a particular recursion level in an n-frequency subdivision of an icosahedron triangle on the sphere may be computed from spherical trigonometry equations if the edges of each sub-triangle are geodesic lines. In the case that one or more edges are not geodesic lines, these edges can be divided into a large number of short segments, and the surface area of the resulting many-sided polygon can be computed using the oriented triangle summation approach developed by Kimerling (1984). When the surface areas of all sub-triangles are known, they can be analyzed statistically to obtain their mean, range, and variance. The range and variance values can be compared with those of the "ideal" partition (range = 0, variance = 0), and the difference in area from the mean can be found for each sub-triangle. These measures allow us to see how close the partitions produced by competing methods approach the equal area ideal.

4 Partitioning On The Icosahedron

Since the twenty triangular faces formed by the twelve vertices of an icosahedron on the sphere are identical in shape and surface area, examining one face is sufficient for understanding how a particular global partitioning method operates. Partitioning involves three basic steps:

1) selecting a cartographic or geometric partitioning method for determining the geographic locations of the vertices for all sub-triangles formed;
2) selecting the n-fold degree of subdivision;
3) selecting the level of recursion.

There are three basic approaches to determining geographic locations of sub-triangle vertices. We term these the map projection, direct spherical subdivision, and polyhedral approaches.

4.1 Map Projection Approach

In the map projection approach, one of the twenty equilateral triangles on an authalic sphere (sphere equal in surface area to the spheroid selected for the earth) is projected as a planar equilateral triangle. An equilateral triangular grid of sub-triangle vertices are then overlaid mathematically on the map projection surface. Each point is back-projected to the authalic sphere, and its geographic coordinates are then transformed to geodetic latitude and longitude using equations for authalic to geodetic latitude conversion (Snyder 1987). Three map projections capable of transforming spherical equilateral triangles into planar equilateral triangles were examined: the gnomonic azimuthal, Snyder equal-area polyhedral, and Fuller-Gray. The Lambert azimuthal equal area projection does not qualify because it does not transform equilateral spherical triangles the size of polyhedra faces into planar equilateral triangles.

Gnomonic azimuthal. This ancient map projection has the well-known property that all geodesic lines are projected as straight lines. If the projection centerpoint coincides with the center of the spherical triangle, its three geodesic edges will be projected as straight lines forming an eqilateral triangle. This property, plus its ease of construction, has led cartographers to use the gnomonic projection for many previous polyhedral world maps. The projection is neither conformal nor equal area, however, meaning that an equilateral triangular grid of sub-triangle vertices across its surface will produce a set of spherical triangles differing in surface area and shape when back-projected to the authalic sphere.

Snyder equal-area polyhedral. This projection which transforms each icosahedron face on the sphere into an equilateral planar triangle while maintaining equivalence of area throughout was devised by Snyder (1992). Snyder's equal-area polyhedral projection is best understood through comparison with a Lambert azimuthal equal area projection of an icosahedron face. The geodesic line edges of the spherical triangle are not straight lines on the Lambert projection, but rather bow outward. Snyder's projection eliminates this by progressively reducing the scale in the direction perpendicular to each edge as we move from the center to the midpoint of each edge. The projection is made equal area by adjusting the scale outward from the center of each edge in the direction parallel to the edge. This results in shape distortion increasing rapidly as each of three lines equidistant from its corresponding pair of edges is approached. These ridges converge to a point of maximum angular deformation at the icosahedron center. This localization of shape distortion along three interior lines is a drawback that we will return to later in the paper. For now, it is sufficient to understand that the largest shape distortion in spherical sub-triangles back-projected to the sphere from a regular triangular partitioning of the projection surface should occur along these three ridges of maximum angular deformation.

Fuller-Gray. R. Buckminster Fuller is well known for his 1943 "Dymaxion" polyhedral world map based on the faces of the cubeoctahedron (Fuller 1983). His later polyhedral world maps, however, were based on the icosahedron, and Fuller's icosahedral projection (Fuller 1982) continues to be used.

Fuller imagined the three edges of each spherical icosahedral triangle as flexible bands curved to lie on the surface of the sphere. Each edge could be subdivided into n equal increments, equally spaced holes could be drilled, and thin flexible bands could be strung between corresponding holes on adjacent edges. This created a triangular network of lines on the sphere, since any combination of three intersecting lines always met at a point on the sphere. Fuller next imagined flattening and stretching the bands into a planar equilateral triangle to create a regular triangular grid of equilateral sub-triangles, the vertices of each being the projection of the corresponding line intersection point on the sphere.

Fuller's idea, although intuitively appealing, was later found physically impossible to achieve. The basic problem is that a regular triangular arrangement of equally spaced parallel geodesic lines on the icosahedral spherical triangle with all line triplets intersecting at points cannot do so when projected to similarly arranged straight lines on a planar equilateral triangle. Each triplet of intersecting straight lines will form a small triangle, whose centerpoint is the best approximation to Fuller's idea. Gray (1995) has developed exact transformation equations for this form of Fuller's projection, differentiable equations suitable for distortion analysis. The Tissot distortion analysis method described in Snyder (1987) was performed on Gray's equations. The range of area distortion is one-tenth that on the gnomonic. The range of angular deformation is far less than on the gnomonic and Snyder projections, although the maximum deformation is the same as on the Snyder. Like on the gnomonic projection, angular deformation increases radially outward from the projection center, and is not localized as on the Snyder projection.

4.2 Direct Spherical Subdivision Approach

The spherical subdivision approach does not employ a map projection, but rather involves direct subdivision of the spherical triangle. A common 2-frequency subdivision method is to find the midpoint of each triangle edge, then calculate the geodesic lines connecting each pair of midpoints. This quarters the icosahedral face, and this spherical quartering procedure can be repeated on the sub-triangles just formed to obtain ever greater numbers of sub-triangles. A similar method can be employed for 3-frequency subdivision, except that the first step is to compute the triangle center, which is the vertex shared by the six central sub-triangles. As with the map projection approach, the last step is to convert the geographic coordinates of all midpoints to geodetic latitude and longitude.

4.3 Polyhedral Approach

The polyhedral approach to icosahedron face subdivision is the basis for geodesic dome construction, pioneered by Buckminster Fuller. We begin with an icosahedron and circumscribing sphere where each icosahedral triangle edge is a chord of the sphere. For 2-frequency subdivision, the midpoint of each edge is raised along a line radiating from the center of the sphere. Four sub-triangles are formed in this manner, and the midpoints of these new chords can again be raised to touch the sphere, creating 16 sub-triangles. This procedure is repeated until the sub-triangles become small enough to closely approximate the sphere, much like plotting a circle as a many-sided polygon. This chord midpoint subdivision method produces a set of triangle vertices identical to those created by halving the geodesic line edges of the spherical triangle, i.e., it is identical to the direct spherical subdivision approach just discussed. However, other of the icosahedron edge chords (dividing into thirds and raising each, for example) will not produce results identical to direct spherical subdivision, as we will demonstrate later.

5 Partitioning Method Comparison

We can now examine the area distortion characteristics of sub-triangles formed by the differing cartographic/geometric partitioning methods at the same frequencies of distortion and levels of recursion. The partitioning methods were examined in their 2- and 3-frequency subdivisions. For 2-frequency subdivision, recursion levels 1-8 were studied, whereas recursion levels 1-5 were studied for 3-frequency subdivision in order to keep the total number of sub-triangles examined roughly similar. The 2-frequency Fuller, Direct Spherical, and Polyhedral approaches produce identical partitionings, as do the 3-frequency Fuller and Direct Spherical approaches. This leaves nine different methods that must be compared in terms of sub-triangle area distortion.

5.1 Area Distortion Analysis Results

Area distortion analysis should determine the spatial pattern of area distortion across the icosahedron face, as well as the statistical properties of areal deviations from the average. We analyze these area distortion characteristics for sub-triangles at different levels of recursion in a partitioning of an icosahedron face. Sub-triangles are analyzed not only because they are the partition units, but also because both the direct spherical subdivision and polyhedral partitioning approaches are not differentiable and hence cannot be subjected to Tissot distortion analysis of map projection equations. Analysis of deviation is sub-triangle surface area on the sphere can be conducted on any partitioning method.

Both spatial and statistical analysis suggest that we standardize the mean area for each level of recursion to 1.0. This both adjusts for the inherently smaller sub-triangle areas with increasingly higher levels of recursion, and transforms the actual sub-triangle surface areas into values that are proportions of the mean.

5.1.1 Snyder polyhedral equal area projection

Notice on the deviation maps (Figure 1) that for 3-frequency subdivision recursion level 3 the icosahedron face is a constant medium gray, representing sub-triangle surface areas of 1.0, a

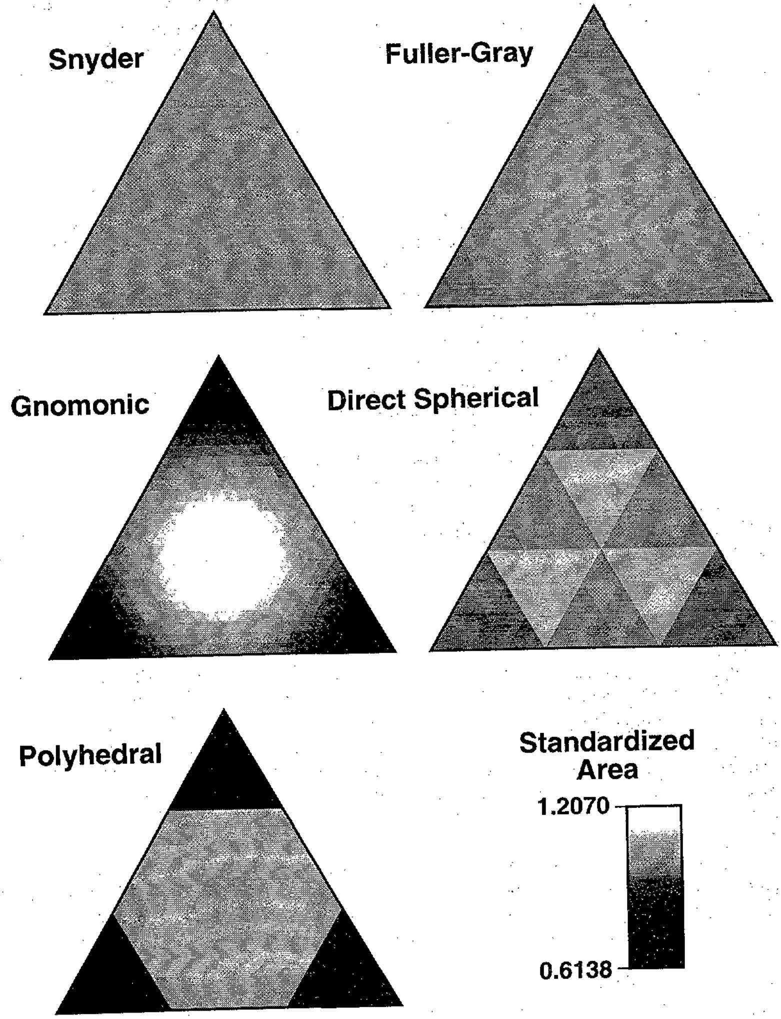

Figure 1. Comparison of partitioning methods for
3-frequency subdivision, recursion level 3.

characteristic of equal area map projections. This is a useful standard for comparison with the other partitioning approaches examined in this study.

Examining a graph showing the range of area deviation for 2- and 3-frequency subdivision, using different partitioning methods (Figure 2), the range in surface area on the sphere is 0.0 for all levels of recursion, another defining characteristic of an equal area map projection. Similarly, a graph illustrating standard deviation in surface area for different methods (Figure 3) shows that for all 2- and 3-frequency recursion levels on the Snyder projection, the standard deviation of sub-triangle area is a constant 0.0.

5.1.2 Fuller-Gray projection

When the equilateral sub-triangles on the Fuller-Gray projection surface are back-projected to the sphere (Figure 1), we see that the spatial pattern of surface area deviation from the mean is different at the beginning levels of recursion, but converges to the same spatial pattern at higher levels of recursion. 2-frequency subdivision produces a central sub-triangle of greater than average surface area, surrounded by three edge sub-triangles of slightly smaller than average area on the sphere. 3-frequency subdivision level 1 results in a hexagon formed by the six central sub-triangles greater than average in area surrounded by three edge sub-triangles of less than average area and lower in area than the 2-frequency edge sub-triangles. This means that the equilateral sub-triangles on the projection surface contain greater surface area on the sphere at the projection center, and progressively less spherical surface area as each corner of the icosahedron triangle is approached.

The statistical nature of standardized sub-triangle areas is one of convergence to stable statistical values with higher levels of recursion. For example, the range of area deviation (Figure 2) begins at 4.4% of the mean for both 2- and 3-frequency subdivision recursion level 1, and converges to a maximum range of 12.4% as sub-triangles become small enough to be computationally indistinguishable from planar triangles. The standard deviation of standardized sub-triangle surface areas (Figure 3) behaves similarly, beginning at 2.2% of the mean and converging to 2.9%.

5.1.3 Gnomonic azimuthal projection

When an equilateral triangular partitioning of the gnomonic projection surface for an icosahedron face, is back-projected to the sphere, the spatial pattern of spherical sub-triangle surface area deviation is similar to the Fuller-Gray projection, as seen in Figure 1. 2-frequency subdivision produces an above average area central triangle surrounded by three below average area corner triangles. Notice though that the amount the central and corner triangles are above and below the mean is far greater here than on the Fuller-Gray projection, or with any other partitioning method examined in this paper.

At higher recursion levels both the 2- and 3-frequency subdivision sub-triangles form a circular pattern of decreasing surface area radially away from the center of the icosahedron face, so that the largest spherical sub-triangle is at the center and the smallest is found at each corner. This circular pattern of sub-triangle surface area follows the Tissot area distortion isolines for the projection.

Sub-triangle standardized surface area statistics for the Gnomonic projection are similar to the Fuller-Gray in that both the range and standard deviation (Figures 2 and 3) increase rapidly at the beginning and reach stable maximum values at higher levels of recursion. The measured range of sub-triangle area is much greater on the Gnomonic projection, by far the greatest range in surface area for any of the partitioning methods examined. Standard deviation values are also higher than with the Fuller-Gray and other methods.

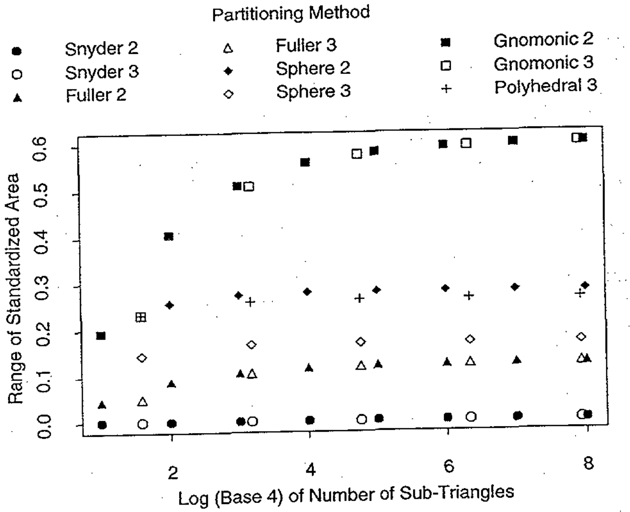

Figure 2. Range of standardized sub-triangle areas for different 2- and 3-frequency subdivision levels of recursion.

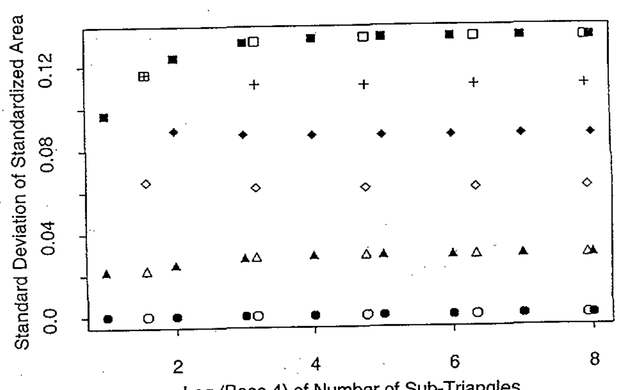

Figure 3. Standard deviation of standardized sub-triangle areas for different 2- and 3-frequency subdivision levels of recursion.

5.1.4 2-frequency direct spherical/ Polyhedral method

Examining the area distortion map (Figure 1) where an equilateral triangle represents the equilateral spherical triangle for an icosahedron face, we see a fractal-like pattern of surface area deviation from the mean. This appears to be similar in form to the fractal surface commonly known as a Sierpinski Gasket (Mandelbrot 1983). The nature of this fractal-like pattern can be seen as we progress from lower to higher recursion levels. Recursion level 1 creates a larger than average central sub-triangle surrounded by three identical sub-triangles of smaller than average surface area. At recursion level 2 the central triangle is quartered into a central sub-triangle larger than its three neighboring corner triangles, although these will also be above the overall mean sub-triangle area for the complete icosahedron face. When each of the three level 1 corner triangles is quartered at recursion level 2, the central sub-triangle will again be larger than the three surrounding corner triangles, although all four might be smaller than the mean surface area.

Statistical analysis of standardized spherical sub-triangle surface area at different levels of recursion reveals similarities and differences with other partitioning methods. Changes in the range of sub-triangle surface area (Figure 2) with increasing levels of recursion behave like the Fuller-Gray and Gnomonic projections -- a rapid increase in the range from recursion levels 1 to 2, with a convergence to a stable maximum range value at higher recursion levels. The maximum range is the second largest among the methods examined, but is closer to the Fuller-Gray than to the Gnomonic range. Standard deviation values (Figure 3), on the other hand, decrease with increasing recursion levels until a stable minimum is reached at higher levels. This minimum is half way between the standard deviation minima for the Gnomonic and Fuller-Gray projections.

5.1.5 3-frequency direct spherical partitioning

Direct 3-frequency subdivision of the icosahedron face into sub-triangles with geodesic line edges produces a fractal-like spatial pattern of standardized sub-triangle surface areas (Figure 1) similar to a 3-frequency fractal-like pattern that is quite distinct from the 2-frequency pattern. We see a three above average surface area center sub-triangles alternating with three slightly below average area sub-triangles. Three below average area corner sub-triangles complete the level 1 partition. This triad pattern of alternating above average area sub-triangles persists at higher levels of recursion, and the entire pattern appears to converge to stability by recursion level 3.

Statistical analysis of sub-triangle spherical surface areas further illustrates similarities and differences between 2- and 3-frequency direct spherical subdivision, as well as the other methods examined in this paper. The range of sub-triangle area (Figure 2) converges to a stable maximum as with the other methods, but the range is consistently lower than with 2-frequency subdivision. The same is true of the standard deviation in surface area (Figure 3), which converges to a stable minimum that is consistently lower than the minimum for 2-frequency subdivision. This lower range and standard deviation is due to the nine sub-triangles formed in the initial 3-frequency partition being closer in surface area than the four initial sub-triangles created in 2-frequency subdivision.

5.1.6 3-frequency polyhedral approach

The spatial pattern of 3-frequency subdivision of the icosahedron face using the polyhedral approach differs from the pattern produced by other methods, but is closest to 3-frequency direct spherical subdivision. Its uniqueness stems from the six identical above average spherical surface area center sub-triangles and three identical below average area corner sub-triangles created by the first level of recursion (Figure 1). This hexagonal grouping of above average area sub-triangles is replicated in the partitioning of level 1 sub-triangles into above and below average area level 2 sub-triangles, and the grouping continues at higher levels of recursion, making the pattern fractal-like.

The replication of six above average and three below average area sub-triangles should produce a large range and standard deviation relative to other methods. Figure 2 shows this for the range, where the polyhedral approach range rapidly stabilizes to a value nearly as high as with 2-frequency direct spherical partitioning, and considerable higher than with 3-frequency direct spherical partitioning. Standard deviation values (Figure 3) rapidly converge to a stable minimum value that is far higher than for either 2- or 3-frequency direct spherical partitioning, a value only exceeded by the Gnomonic projection.

6 Conclusion

We have conducted a comprehensive analysis and comparison of several partitioning methods for a face of the spherical icosahedron, anticipating that the analysis methods developed here can be applied to other partitioning approaches. The numerical comparison methods developed here are closely linked to survey sampling requirements for a global grid, as expressed in the Goodchild criteria. We focused on the nature of surface area and compactness variation when identical frequencies of subdivision and levels of recursion were applied to five basic partitioning methods. The area and compactness characteristics we discovered at higher recursion levels are if particular importance since a global sampling grid will likely be developed and implemented at the highest levels of recursion examined here, or even higher.

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