A MATHEMATICAL MODEL FOR PRINTING COLOR REPRODUCTION BASED ON UNDISCRIMINATION DOT GAIN NATURE

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Abstract

This paper analyzes two kinds of dot gain in detail and develops a mathematical model for printing color reproduction based on undiscrimination dot gain nature. The experiment shows that the model is of simple caculation, high precision and easy modification.

1. Introduction

At the present time there are two kinds of methods to establish the relation between the dot area and the chromaticity value: one is to set up a data—base that maps the dot area to the chromaticity value. [1,2,3] It is a black box method used by many researchers. The other is to find a mathematical model about the relation according to the mechanism of the halftone color formation. The method microscopically dissects the geometrical and optical features of dots, and macroscopically seeks for the chromaticity values (eg. CIE1931—XYZ tristimulus values) of the color chips based on the colorimetry theory. The method also approximately resolves the dot area from XYZ by the differential calculus. Most researchers adopt the Neugebauer equations to establish the model. In the equations the effective dot area (EDA) is an important variable that is closely related to the geometrical dot gain (GDG) and optical dot gain (ODG). In fact, it is a difficult problem to accurately discriminate GDG and ODG in measured color data. [4] This paper develops a new method, a mathematical model based on undiscriminating dot gain nature which is illustrated in COLOR ATLAS (four—colors printing part). The experiment and its conclusions in paper can be spread to the situation of any other primary colors and their overlap.

2. About Neugebauer equations

In 1939, Neugebauer presented a physical model of color formation by halftone dots as follows:

$$X = \sum_{i=1}^{9} A_i X_i; \quad Y = \sum_{i=1}^{9} A_i Y_i; \quad Z = \sum_{i=1}^{9} A_i Z_i \tag{1}$$

Where X, Y, Z are the tristimulus values (TVS) of a chip; X_i, Y_i, Z_i are the TVS of each fractional color in a unit area. A is the fractional area. Neugebauer's physical model of color formation by halftone dots bases on the light paths showed in Fig. 1. [7] By Neugebauer's view, the dot domain encompasses four light paths, each is an independent color formation "

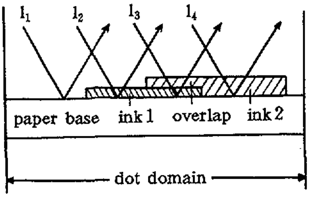

Figure 1. Capturing essentials of Neugebauer's physical model of color formatiom by halftone dots.

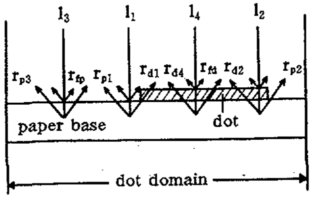

Figure 2. Reflectance behavior due to the base scattering.

event". So predicting A can be viewed as equivalent to determining the probability of randomly touching a particular color on the domain. The calculating formulas can be look at reference 3.

Obviously, finding EDA is a key that Neugebauer's equations reflect color spaces transformation accurately. Here using the term "effective dot area(EDA)" is purposely to differ from the area, "label dot area (LDA)", labeled in COLOR ATLAS. The former indicates the dot area that is directly or indirectly measured by optical instrument. For a variety of reasons, the two kinds of dot areas have fairly large difference.

3. Dot gain and EDA

There are two reasons that the EDA differ from the LDA: (1) The dot gain in geometric size results from physical or mechanical reasons in printing process. It is the geometric dot gain (GDG). (2) In color measuring process, partial absorption and scattering of the incident light of optical instrument by base and ink makes the reflectance (from a halftone dot pattern) received by the instrument so little that the dot area seems larger than its real area. It is the optical dot gain(ODG). The measured data of chips have simultaneously included the two kinds of the dot gain factors which must be considered in finding EDA.

For a particular printing product, its amount of GDG has been fixed, but its amount of ODG will change with the light source characteristic of the colorimetric instrument, the illumination ——observation condition, and the wavelength interval of the sensor. Fig 2 shows the reflectance behavior due to base scattering for a unit area halftone pattern of dot area A and base area (1−A). Its incident light is a single band light. The four incident light rays (l_1, l_2, l_3, l_4) have four different reflection paths by different incident positions. The r_{pi} indicates the reflectance of incident light l_i reflected from the base; the r_{di} is the one reflected from dot; r_{fd}, r_{fp} are incident Fresnel reflection of the surface of ink and base respectively. Clearly, only l_1 and l_2 casuse dot gain. Let t_d = the dot ink's transmittivity, S_{pd} = the fraction of the light scattered from the base into the dot area, S_{dp} = the fraction of the light scattered from the dot area into the base, and R_t = the total reflectance in a unit area. So

$$R_t = (1-A)r_{fp} + (1-A)(1-r_{fp})r_p(1-S_{pd})$$

$$+ A(1-r_{fd})r_p t_d S_{dp} + A r_{fd} + A(1-r_{fd}) t_d^2 r_p (1-S_{dp})$$
$$+ (1-A)(1-r_{fp}) r_p t_d S_{pd} \tag{2}$$

Taking the boundary conditions into consideration, when A = 0, letting R_0 = the total reflectance; when A = 1, letting R_s = the total reflectance, so

$$R_t = (1-A)R'_0 + AR'_s \tag{3}$$

where: $R'_0 = R_0(1-S_{pd}(1-t_d)) + r_{fp}S_{pd}(1-t_d)$ (4)

$$R'_s = R_s(1+S_{dp}(1-t_d)/t_d) + r_{fp}S_{dp}(1-1/t_d) \tag{5}$$

From Eq. (4), (5), it is known when A = 0 or 1, $R'_0 < R_0$ and $R'_s > R_s$. So, the physical implication of Eq. (3) is that the dot size has not changed, but rather the reflectance of the base and dot has changed to R'_0 and R'_s respectively. It means that the effective density of the base has increased, and of the dot has decreased. If $S_{pd} = 0$ and $S_{dp} = 0$ ie the scattering phenomenon not been considered, $R'_0 = R_0$, $R'_s = R_s$. So Eq. (3) becomes Eq. (6), it is the famous Murray—Davies equation:

$$A = \frac{R_0 - R_t}{R_0 - R_s} = \frac{10^{-D_0} - 10^{-D_t}}{10^{-D_0} - 10^{-D_s}} \tag{6}$$

It proves that Eq. (6) is true only on unscattering condition. The condition will be possessed only if the base is an ideal mirror reflector, while $R_0 = 1$ ($D_0 = 0$). To include the influence of the base scattering factor, Yule and Nielson put the n factor which demonstrates the base behavior into Eq. (6). The result is given in Eq. (7):

$$A = \frac{1 - 10^{-D_t/n}}{1 - 10^{-D_s/n}} \tag{7}$$

For the same reason, Pobboravsky and Pearson modified Neugebauer equation as follows: [4]

$$X^{1/n_X} = \sum_{i=1}^{9} A_i X_i^{1/n_X}; \quad Y^{1/n_Y} = \sum_{i=1}^{9} A_i Y_i^{1/n_Y}; \quad X^{1/n_Z} = \sum_{i=1}^{9} A_i Z_i^{1/n_Z} \tag{8}$$

In Eq. (7), (8), to know n or n_X, n_Y, n_Z, we must first to know A or A_i. The A or A_i is the geometric fractional dot area. So if the two equations are used, the influence results of GDG and ODG must be discriminated. But it is difficult for the color data measured. [8] Some researchers have tried to get GDG and thereby to get n value by physical method. [12] Some researchers do that through mathematical method. [5,13] But those researchers all found that though base, ink and lines per inch are fixed, the n value is not a constant. The reason is that the fact $R_0 = 1$ is still premised in Eq. (7), and the scattering between base and dot is not considered. [8,9] The n value is closely related to printing condition and printing product quality. On the situation, the n value is not likely to be a constant. Using Eq. (7), (8) is limited.

4. A transformation model of undiscriminating dot gain nature

In the last section, we presume that the incident light is a single band. If it is a wide band light, the mechanism of ODG will become very complex since the base and the ink absorb and reflect various wavelength light selectively. For tristimulus—filter colorimeter, its incident light comes from a standard light source (wide band), its sensors receive the lights in red, green, and blue spectrum respectively. The responses of the three photocells or photomultipliers in the instrument are proportional throughout the visible spectrum to the CIE 1931 color—matching functions, or to some other linear combination of them. With an exact duplication of the CIE color—matching functions, the responses of these photocells give TSV of the radiant power incident on the photocells. [10] Thus, the TSV of the measured color under a specific source shows the proportion of red, green, blue spectrum reflection to the total reflection. Therefore the factors of ODG for X, Y, Z values are different. This important conclusion shows why the n value in Eq. 8 be modified, and incite this conclusion: While GDA is constant, the EDAs corresponding to X, Y, Z values are different.

Suppose that the EDAs corresponding to a color's TSV (X_t, Y_t, Z_t) are (a_X, a_Y, a_Z). The TSVs of the single—color ink are (X_s, Y_s, Z_s), and of the base are (X_0, Y_0, Z_0). According to the Grassman additive law, the equation as follows are tenable:

$$X_t = a_x X_s + (1 - a_x)X_0; \quad Y_t = a_Y Y_s + (1 - a_Y)Y_0; \quad Z_t = a_Z Z_s + (1 - a_Z)Z_0 \quad (9)$$

Thereby Eq. 10 is obtained:

$$a_x = \frac{X_0 - X_t}{X_0 - X_s}; \quad a_Y = \frac{Y_0 - Y_t}{Y_0 - Y_s}; \quad a_Z = \frac{Z_0 - Z_t}{Z_0 - Z_s}; \quad (10)$$

Eq. 9 and 3 have the same forms but the different implication. Eq. 3 implicates that the dot area has not changed but the reflectance of the base and the dot. Eq. 9 implicates that the TSVs of the base and the dot do not change but the EDA do. Please note that this EDA, consisting of GDG and ODG, is as a result of comprehensive influances by the two kinds of the dot gains. Therefore using Eq. 10 does not need to discriminate the two kinds of the dot gain, and to seek n value, only need to calculate a_X, a_Y, a_Z values, then to do the fractional areas A_{iX}, A_{iY}, A_{iZ} (i=1, 2, ... 9) corresponding to X, Y, Z values from the equations in table 1, and finally to do the TSV of the chip from Eq. 1. Thus to solve the TSV from LDA is to solve a_X, a_Y, a_Z values from LDA. The problem is easy to solve. Firstly the X, Y, Z values of each primary color's halftone scales (include $X_0, Y_0, Z_0, X_s, Y_s, Z_s$) is measured, then a_X, a_Y, a_Z from Eq. 10 is calculated, the regress relation between LDA and a_X, a_Y, a_Z values for each scale is established. Two orders regress model usually achieves a high precision.

A part of data mearsured from COLOR ATLAS is used to check the precision of the model. The data shows that a_X, a_Y, a_Z values of the same color have a great difference. The maximum is just the EDA which spectrum complements with the ink's. The reason is that the scattered light from the base to the dot has greatly been absorbed, it makes the EDA increased. The minimum among the a_X, a_Y, a_Z is the EDA which spectrum is the same as the ink's. In this situation, the scattered light from the base to the dot has greatly been transmited through the ink layer. If the ink was ideally the same color as the spectrum of X, Y or Z, the minimum of EDA will be GDA of the dot. But it is not so ideal in fact.

On the pages of two—color combination, the largest color difference is 6.35 NBS, the colors that $\Delta E > 5.0$ NBS are 2.8% of total colors on each page. On the pages of three—color combination, let yellow halftone color be as the base color to establish the model, a good result is obtained too. The colors that $\Delta E > 5.0$ NBS are 4.6% of total colors on these pages. The facts above prove that the model simulates the changing rules of the dot gain and the TSV on each page of the atlas. The accuracy of the model for all pages of the atlas will depend on the primary color, its data, the data error and the printing quality. These elements are decisive not only for the data—base setting method but also for the modelling method.

The descriptions above are the process of transformating from LDA to TSV. Based on the process, the transformation from TSV to LDA can be completed by iteration. In the inverse transformation, there are four combinations of three inks for a given TSV ;.(Ye, Ma, Cy), (Ye, Ma, Bk), (Ma, Cy, Bk), (Ye, Cy, Bk). Try all four possibilities, one after another, reject the unreasionable answers. Pay attention to whether the given TSV go beyond the printing gumat. If the color is beyond the gumat, a resolution for the two situations must be given; When the original data point is far away from the nearest point in the gumat and the distance is larger than the limited color difference, iteration will be given up, it shows that the color can not be reproduced by printing; when the original data point is near the gumat and the distance is in the limited color difference, the original data will firstly be preprocessed by cliping or compressing method to find a substitution for the data, then iternation will be completed smoothly.

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