THE NEW ADAPTIVE CONFORMAL CARTOGRAPHIC PROJECTION AND ITS COMPARISON WITH OTHER CONTEMPORARY PROJECTIONS UNDER THE CONDITIONS OF A PERMANENT INCREASE OF THE QUANTITY AND QUALITY OF CARTOGRAPHIC DATA

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ABSTRACT

The paper gives the complete theory of the recently developed adaptive conformal cartographic projection in its several variants, which is superior, according to the criterion of minimum of linear deformations for the mapping region, to all contemporary cartographic projections known.

Based upon the originally derived partial differential second order equations, describing the conformal mapping of a rotational ellipsoid onto a plane, as well as upon the contour conditions formulated according to Chebyshev - Grave's theorem, an iterative method for the determination of the linear scale function has been elaborated. The function has been sought in the form of a harmonic polynomial, the coefficients of which have been defined after the least square method.

In addition, the paper presents a methodology for finding the optimal CAMPREL[1)] projection, which simultaneously satisfies the criteria of both the minimax and the variation types. The optimal projection has been chosen among several thousands of projection variants by means of a multi-criteria analysis. Thereby, a uniform and symmetric distribution of the extreme linear deformations represents a higher order criterion, which is used to supplement the other criteria.

In the present times of a sudden development of the possibilities of micro computers, and of the increasing necessity for a development of Geo-information systems (imposing a permanent progress in the quality and quantity of cartographic data not only of the countries, continents and oceans on the Earth, but also of other celestial bodies), a comparison of CAMPREL optimized projection with that of UTM and Lambert's projections for small countries (the former Yugoslavia), as well as for the large ones: Urmaev's projection for the European part of the former USSR, Frankich's projection for Canada, and Snyder's for USA, has undoubtedly confirmed the superiority of the optimized CAMPREL projection.

1. INTRODUCTION

The traditional conformal cartographic projections we use for nearly hundred years, were developed at demands of the last century, under conditions of that time, with logarithmic tables representing the only computing device. Some of the traditional cartographic projections were optimal for the production of military topographic maps for use in the old artillery; the other ones were optimal for the large scale maps. All of them have many good features, but from the contemporary viewpoint it would be hard to claim that traditional cartographic projections are optimal today.

[1)] Conformal Adaptive Mapping Projection of the Rotational ELlipsoid.

Instead of opening logarithmic tables, now we have Notebook Personal Computers, based on Intel - 80486 or Pentium microprocessors, winded up to 150 Mhz, with 1.2 GB Hard Disk and Compact Disk Drives, quite enough for read/write GIS-data bases, thematic maps and massive read-only geographic atlases.

The example of a new Computer Assisted Cartographic products is a whole range of "softcopy" electronic maps, generated from related GIS-information, displayed on the stationary 16"÷26" Color Monitors, connected with a Notebook PC in the appropriate Docking Station.

Geographic territories displayed by the electronic maps can be very large, and the required accuracy can be very high. The eight-step zooming technique resolves all problems when, for example, a map of the entire African continent and a detail of the Egyptian Ramzes II temple, are displayed on the same monitor. But, if the highest possible map-accuracy is required, the parameters of cartographic projection, i.e. the mathematical background of maps generation, cannot be the same in all zooming steps.

Very heavy demands are imposed on the modern cartographic projections in the basis of electronic maps and other Computer Assisted Cartography products, accurately describing not only our planet Earth, but also the Moon, space planets and their satellites [1]. Disregarding such demands, the most important and most severe demand, which includes all other demands, still remains in the domain of classical cartography. That primary demand reads:

A contemporary conformal cartographic projection must provide for not only the minimal inevitable linear distortions for the maximally possible mapping region, but it also must provide that the minimal distortions are uniformly and symmetrically distributed over the mapping region, in all locations and all directions.

2. THE NEW OPTIMAL CARTOGRAPHIC PROJECTION "CAMPREL"

The new optimal cartographic projection with the working title "CAMPREL" (Conformal Adaptive Mapping Projection of the Rotational ELlipsoid) was developed in the Institute of Geodesy, University of Belgrade, as doctoral dissertation defended on July 1992 by Ivan Nestorov, B.Sc., M.Sc. [6].

The basic concept of the new optimal cartographic projection CAMPREL was defined by choosing the "reverse assignment method" in mathematical cartography instead of the traditional "direct assignment method". According to the "reverse assignment method", a new cartographic projection can be developed if the functions of the projection characteristics are known; then the functions of direct mapping and inverse mapping can be determined. According to the "direct assignment method" in mathematical cartography, a new projection can be developed if the mapping functions are known; then the functions of the distortion characteristics can be determined. The "reverse assignment method" requires more sophisticated mathematics, but the advantages are: more generalized solutions and the fact that mapping functions and functions of the distortion characteristics can be defined non-analytically by their numerical values in random locations of the entire mapping region.

The conformal mapping of the rotational ellipsoid onto a plane can be described by Poisson's differential equation: [6]

$$\frac{\partial^2(\ln m)}{\partial q^2}+\frac{\partial^2(\ln m)}{\partial l^2}=\left[\frac{1-e_P^2\cdot\sin^2(\varphi)}{1-e_P^2}\right]\cdot\cos^2(\varphi) \tag{1}$$

where:

q and **l** are isometric coordinates on the rotational ellipsoid;

φ is geographic latitude; **m** - linear scale function, and $\mathbf{e_p}$ - *eccentricity* of meridian ellipsis

Laplace' differential equation, describing conformal mapping of the rotational ellipsoid onto a plane:

$$\frac{\partial^2(\ln\nu)}{\partial q^2}+\frac{\partial^2(\ln\nu)}{\partial l^2}=0 \quad , \quad \nu=m\cdot\frac{\cos(\varphi)}{\sqrt{1-e_P^2\cdot\sin^2(\varphi)}} \tag{2}$$

Differential equations (1) and (2) are very important in the theory of conformal cartographic projections. By the integration of these equations many known and many as yet unknown conformal projections can be obtained.

The second basic assumption in formulating the new CAMPREL projection can be described by the Tchebyshev-Grave theorem[2)]:*Among all conformal projections, the best one, with minimal value of the ratio S_c (S_c=max.lin.scale / min.lin.scale) for the entire mapping region, will be that which provides the constant value of the linear scale logarithm on the contour of the mapping region.*

Accordingly, the linear scale function of the optimal conformal projection can be obtained by solving the Poisson's differential equation (1) under the boundary condition:

$$\ln(m)\big|_{\Gamma}=\ln(K) \tag{3}$$

or by solving the Laplace' differential equation (2) under the boundary condition:

$$\ln(\nu)\big|_{\Gamma}=\ln\left[K\cdot\frac{\cos(\varphi)}{\sqrt{1-e_P^2\cdot\sin^2(\varphi)}}\right] \tag{4}$$

where Γ is the contour of the mapping region, and **K** is value of the constant linear scale, assigned along the mapping region's contour.

The solution of the optimal mapping of the rotational ellipsoid onto a plane, formulated in this way, represents a generalization of Urmaev's solution, who provided the mapping of a sphere onto a plane. [10]

The contours of the mapping region can not be defined analytically; therefore, the approximate numerical methods are the only way of solving the differential equations (1÷4). However, the approximate solution can be very close to the exact solution if modern computers are used.

There are two groups of numerical methods. To the first group belong all methods requiring the approximation of the wanted function, but the contour conditions are completely fulfilled in all points. To the second group belong all numerical methods requiring exact definition of the harmonic functions, but the contour conditions are fulfilled only approximately. A well-known method from the second group is the Least Square Method.

The solution of the equation (2) under the boundary condition (4) can be obtained in the form of the k-order symmetric harmonic polynomial:

$$u_k=\sum_{j=0}^{k} a_j\cdot P_j \tag{5}$$

or in the form of the k-order non-symmetric harmonic polynomial:

$$u_k=\sum_{j=0}^{k}(a_j\cdot P_j+b_j\cdot Q_j) \tag{6}$$

[2)] The theorem has been formulated by the Academician P.L.Tshebyshev in 1856, and the proof was given by the Academician D.A.Grave in 1896. The less sophisticated proof of this theorem was given by J.Milnor in 1969 [5].

where **P** and **Q** represent the real and the imaginary term of the k-order harmonic polynomial:

$$u_k = (q + i \cdot l)^k \; ; \; i^2 = -1 \tag{7}$$

To the polynomial (7) applies the Morozov's recurrent formula:

$$\begin{array}{ll} P_0 = 1 & P_{j+1} = q \cdot P_j - l \cdot Q_j \\ Q_0 = 0 & Q_{j+1} = q \cdot Q_j + l \cdot P_j \end{array} \tag{8}$$

where a_j and b_j (j=1,2,...,k) are unknown coefficients. The unknown scale factor function can be expressed in the form:

$$m = e^{\varepsilon_S} \tag{9}$$

$$m = e^{\varepsilon_A} \tag{10}$$

where **e** is the base of natural logarithms:

$$\varepsilon_S = a_0 \cdot P_0 + a_1 \cdot P_1 + a_2 \cdot P_2 + \ldots + a_k \cdot P_k - \ln\left[\frac{\cos(\varphi)}{\sqrt{1 - e_P^2 \cdot \sin^2(\varphi)}}\right] \tag{11}$$

$$\varepsilon_A = \varepsilon_S + b_0 \cdot Q_0 + b_1 \cdot Q_1 + b_2 \cdot Q_2 + \ldots + b_k \cdot Q_k \tag{12}$$

The last equation (12) can be formulated for n discrete points on the contour of the mapping region (n ≥ k). By the least square method we can then determine the a_j and b_j coefficients. Then, with the known coefficients a_j and b_j we can determine the linear scale's values in all the points inside the mapping region, so that the linear scale function of optimal mapping is completely defined.

In the case of conformal mapping we have m=n and the orthogonal net of coordinate lines ($\Theta = \pi/2$), so the three characteristics of a new projection are defined by the equations (1)÷(12). The fourth characteristic γ can be determined from the differential relations:

$$\gamma_\varphi = \frac{\mu_\lambda}{\nu} \quad , \quad -\gamma_\lambda = \frac{\nu_\varphi}{\mu} \quad , \quad \nu = m \cdot \frac{\cos(\varphi)}{\left[1 - e_P^2 \cdot \sin^2(\varphi)\right]^{1/2}} \quad , \quad \mu = m \cdot \frac{\left(1 - e_P^2\right)}{\left[1 - e_P^2 \cdot \sin^2(\varphi)\right]^{3/2}}$$

In the conformal mapping theory these relations represent the basic system of differential equations.

If we know the four independent mapping characteristics in every point inside the mapping region, the complete mapping is defined [4]. Knowing the four mapping characteristics, we can determine the equations of the direct and inverse mapping of the rotational ellipsoid onto a plane.

3. COMPUTER GENERATED VARIANTS OF THE "CAMPREL" PROJECTION AND THEIR COMPARISON WITH OTHER OPTIMAL CONTEMPORARY PROJECTIONS

The "minimum of distortions" over the entire mapping region can be defined according to Airy's criterion and the criteria of Jordan, Airy-Kavrayski and Jordan-Kavrayski [2,3,4]. Beside this numerical criteria, the new numerical criteria are defined: the minimum of the diapason of linear distortion's absolute values; the minimum of the relative change of linear distortion's extreme values; the minimum of the relative change of the logarithm of linear distortion's extreme values.

Developing new projections which meet the numerical criteria for the "minimum of distortions" leads mathematically to solving the variation assignment with a conditional extreme. Therefore, these criteria are named variation criteria.

Based on the algorithm for numerical integration of the partial differential equations of the second order, a new computer program is developed for the generation of new CAMPREL variants by the least squares method. The input data for the computer generation of the new symmetric variants are: the geographic longitude of boundary parallels and meridians; the degree of the harmonic polynomial (5). Based upon that polynomial the linear distortion's constant value along the contour of the mapping region are derived. The input data for the computer generation of the new non-symmetric variants are the same, except the arbitrary location of the points on the contour of the mapping region.

By varying the number of the contour points; the degree of harmonic polynomial (5) and linear distortion's value along the contour of the mapping region, we obtain many different variants of the new projection for the given mapping region. For every CAMPREL variant, our program calculates the projection quality indexes: Jordan's total distortion; Jordan-Kavrayski's total distortion; the minimal linear distortion; the maximal linear distortion; the maximal relative change of the linear scale; the diapason of the linear scale changes; the change of linear scale's absolute values; the rela–tive change of linear scale's logarithm; the variance of the failure to satisfy the boundary conditions. For every CAMPREL variant, the program generates the table of linear scale's distribution, as well as the map of isocoles - the lines of equal linear scales over the mapping region.The best CAMPREL variant is defined by key-sorting according to variation criteria.

4. SAMPLES

The new procedures for computer generation of the optimal conformal adaptive cartographic projections has been tested on the territories of small countries (former Yugoslavia) and large countries (former USSR, Canada and USA).

The optimal CAMPREL symmetric projection generated for the territory of the former Yugoslavia (Boundary conditions: 54 contour points; λ_W=13.25, λ_E=23.25, φ_S=40.75, φ_N=47.00) and compared with the traditional Lambert's or UTM-projections, yields much better quality: the total distortions distributed uniformly in all directions with their absolute values 60%÷250% less than the distortions in the UTM-projection [6].

For the territory of the European part of the former USSR the new CAMPREL projection yields the total distortions in the symmetric diapason (from -0.085 to +0.0087) with the absolute values 50% less than the distortions in the Urmaev's projection [10] from 1947. (Fig. 1)

For the territory of Canada the new CAMPREL projection with fourth degree of the harmonic polynomial yields the total distortions in the symmetric diapason (from -0.0155 to +0.0163) with the absolute values 8% less than the distortions in the K.Frankich's projection[3)] [2] from 1982 (Fig. 2).

The optimal CAMPREL symmetric projection generated for the territory of USA, excluding Alaska and Hawaii (Boundary conditions: 36 contour points; λ_W=125.00, λ_E=65.00, φ_S=25.00, φ_N=49.00) yields the total distortions in the symmetric diapason (from -0.0122 to +0.01225) with the absolute values approximately the same as in the J.Snyder's minimum-error projection [8] from 1984 (Fig. 3).

[3)] Dr K.Frankich creates in Canada, but some of his scientific works can be considered as the continuation of the Russian school of mathematical cartography.

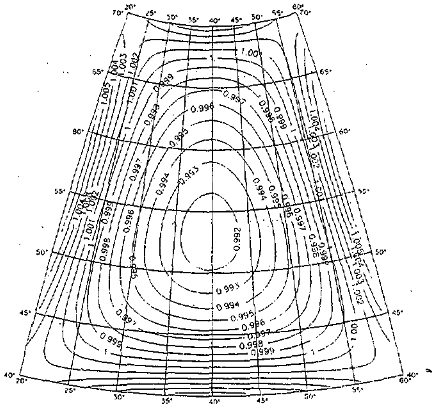

Fig. 1 The map of isocoles of the "CAMPREL" symmetric projection, generated for the territory of the European part of the former USSR.

The optimal function of the linear scale is the ninth degree harmonic polynomial.

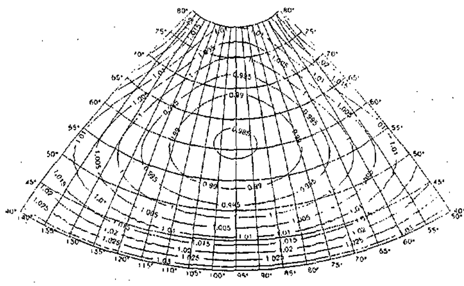

Fig. 2 The map of isocoles of the "CAMPREL" symmetric projection, generated for the territory of the Canada.

The optimal function of the linear scale is the fourth degree harmonic polynomial.

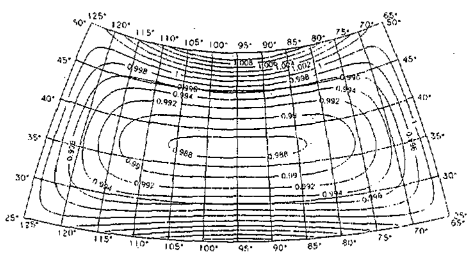

Fig. 3 The map of isocoles of the "CAMPREL" symmetric projection, generated for the territory of the USA, excluding Alaska and Hawaii.

The optimal function of the linear scale is the ninth degree harmonic polynomial.

5. CONCLUSIONS

The results obtained in our research shows that the scientific works belonging to the Russian school of cartography, those from the past century (Tchebyshev, Grave), as well as the contemporary ones (Urmaev, Mescheryakov, Frankich), can excellently provide the mathematical background for computer generation of the new optimal conformal cartographic projections.

Under the contemporary conditions of a steady increase of the quantity and quality of cartographic data describing not only the small and large countries, continents, seas, islands and oceans on the Earth, but also describing the Moon and other celestial bodies, this newly developed conformal cartographic projections optimally fulfill the very strict demands imposed on the modern cartographic projections in the basis of all electronic maps and other Computer Assisted Cartography products in the present epoch of GIS and expert-systems developed and implemented in different domains of activities all over the world.

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