

# REVISITING THE PROJECTIVE PROPERTIES OF HISTORIC NAUTICAL MAPS OF THE MEDITERRANEAN AND THE AEGEAN

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## Summary

Analytical methods for the investigation of the possible projective properties of historic nautical maps with emphasis on the Mediterranean and the Aegean basin representations are proposed especially in view of the new digital processing techniques offered by the raster-wise analyses. A general layout for the design of the problem is presented and alternative scenarios are used in order to detect some properties which could be used in order to draw conclusions or indications related to the projective origin or the projection compliance of the historic nautical maps.

## Introduction

The projective properties of the late medieval and the early and mid-renaissance nautical maps, is *per se* a proper topic in the specialised literature on the map-history of that period. The issue is indeed not new (Fiorini, 1881). Many theories were developed on the issue (Campbell, 1987) and experimental models have been tested (Tobler, 1966; Mesenburg, 1987) in order to approach, understand, explain or approximate the projective nature of the cartographic background of these fascinating maps. In these studies, the driving force in investigating the projective origin of the nautical maps' background could be associated to mainly two factors namely to the impressive accuracy of the coastline depictions and to the continuous propagation, through the ages, of navigation knowledge and practice.

This field of analysis is also associated to the study of the deformational patterns of old maps when compared to the modern counterparts (Koussoulakou et al, 1983; Badarioti, 1987; Mekenkamp, 1990; Livieratos et al. 1999). In a certain extend, this specialised research is now a part of the History of Cartography indeed not yet definitely concluded. In the literature many scenarios have been reported covering a broad spectrum of hypotheses supported either by theoretical intuition or by analytical

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evidence. The main stream of approaches to the problem on the projective nature of the nautical maps use as models cylindrical, conic or azimuthal equidistant projections and their variants.

From all analyses carried out it is not indeed possible to draw a definite conclusion on the proper projection to be assigned to the nautical maps. The various conclusions are controversial and do not formulate a globally consistent theory on the matter. The spectrum of possible answers to the question on the projective background of the nautical maps, of the XIV, XV and XVI centuries, varies from the position of the total absence of a projective background to the support of specific map projections governing the geometric parameterisation of such maps. The outcome is such contradictory that it is not possible, for the moment, to arrive to a definite, consistent and generally accepted theory on the matter. Of course and in a certain extent, it is possible to classify the "reliability" of the conclusions obtained by each of the methods used in order to treat the problem, if this method is subject obeying some rigorous analytical rules. Conclusions with no analytical foundation are equally appreciated and highly valuable in the context that they allow the understanding of the general or specific historic conditions, which could -or could not- influence the introduction of a certain projection in designing the nautical maps. But from the experimental point of view, these conclusions are lacking the evaluation criteria, which are necessary in order to derive and control a possible solution to a mathematical problem (as the map-projection is) using analytical tools, which fit the mathematical problem. This paper deals with classes of old nautical maps from the XIV, XV, and XVI centuries representing the Mediterranean and the Aegean Sea.

### **The new digital approach**

The main approaches to the problem concerning the projective properties of the nautical maps are facing issues related to the processing difficulties in this field of analysis due to the requirement of huge amounts of complex analytical computations and numerical tests. This is necessary in order to obtain results from a great number of cases satisfying a variety of types of such maps made in different epochs of time. The difficulty in handling such huge amount of numerical data and the associated algorithmic weight were most probably the main reasons of a rather poor devotion on this particular field of research.

Today, the problem seems to regain its old fashion and thrill. Thanks to the powerful new digital tools, properly designed for the analysis and transformation of maps in *raster-form*, the study of the projective background of the historic nautical maps becomes a friendly, easy to handle and fast process. In addition, the new raster-wise digital approaches are combined and integrated with relevant graphics offering for this purpose, a unique processing and *visual control environment*. This integration allows alternative testing, becoming, thus, the whole issue of old maps analysis indeed attractive and highly productive. In this context, it is worthwhile to try relevant research

in order to revisit the task of the old map projection and deformations (see e.g. Guerra et al., 1999; Boutoura, 2000).

The scheme proposed here is referred to some maps representing different schools of mapmaking. They are transformed in digital raster-form and alternative best-fitting processes are used in a unified processing and visual environment, which permits the on-line control of both the graphical and the numerical parameters of the fitting. The results allow some possible classifications and the study of deformations analysis (Boutoura and Livieratos, 1986) in linear, and rotational sense. The whole treatment is seen as a contribution in the field of analysis of old maps giving new impulses thanks to the new integrated digital processing technologies, which fit the nature of the problem.

### **The projection-fitting approach**

The *projection-fitting* approach in treating the projective properties of old maps is formulated in the following way: “Given a map  $m'$  with unknown projective properties (i.e. the old map) find to which map  $m$  of known projective properties fits best”. This process usually requires three things:

- a. A set of properly distributed common points recognisable in both maps,
- b. A transformational machine, which brings  $m$  and  $m'$  into a best fitting relation, and
- c. The choice of an evaluation criterion on which the final acceptance of the best fitting is based.

In the first case of requirements, the common points are used as control points in the transformation process. Here, the projection coordinates of the control points of  $m$  are compared to the plane coordinates of the  $m'$  as obtained by digitalisation. The transformation normally used is the Helmert's model, which safely preserves the actual figure of  $m'$  (i.e. similarity) allowing only global alterations in the position of the reference system origin, in the rotation and in scale. The transformation translates the digital coordinates of  $m'$  into the projective coordinates of  $m$ .

The criterion for the acceptance of the fitting is usually the magnitude of the scalar *standard error of the residuals*. But it seems that it should be more realistic to use as criterion, the *spatial distribution of the residuals* since it gives a pattern of higher or lesser areas of discrepancies between  $m'$  and  $m$ . As far as the old nautical maps in question are concerned here, this spatial pattern can also be related and interpreted with respect to the maritime activity in the sea areas represented in  $m'$ .

It is evident that the above procedure presupposes the alternative testing of various  $m$  with different projective properties. This was once an indeed heavy computational task but it is an easy routine today, thanks to the raster-wise digital technologies available. Maps  $m$  and  $m'$  are digitised raster-wise and/or vector-wise and introduced into widely spread software environments which support *graphics* as well, like the MatLab package in association with I/RAS C “image-to-image” process (Balletti 2000) or the combination of Projection Manager with I/RAS C for an “image-to-map” process (Boutoura 1999, 2000). This approach is a type of analysis in which the projection

under investigation is postulated. In other words it is an a-priori assignment of the projective properties.

A by-product of the above best fitting process and of the designation of the projective properties of  $m'$  is the a-posteriori *deformation analysis* of  $m'$  which can be carried out as far as the global or the local deformations are concerned with respect to linear and/or angular deformational components.

In many cases, *visualisation* indicates the proper path to follow, in association to the computed residuals. A "trial and error" approach is sometimes the only realistic way to tackle the problem regulated by the researcher's expertise in map projections. This is easier today, using the efficient software for the raster treatment of the map file transformations both computationally and visually in the same digital environment. The advantage of working in a unique digital environment is focused mainly on the simultaneous visual control, which follows the computational fitting process. In this way the "trial and error" control is visually accepted or rejected leading to a convergence on the final fitting approximation.

### **The regression approach**

The approach first introduced by Tobler (1966) is based on a *bi-dimensional regression* model due to which digitised plane orthogonal coordinates of selected well recognised and well distributed points on  $m'$  –control points– are brought into linear correspondence with their geographical coordinates as derived from their known values from any actual map  $m$ .

According to this model the unknown parameters of the regression model are computed via a regular least squares adjustment thus, a continuous field of displacements can be computed illustrating the distortions of  $m'$  with respect to the geographical parameterisation. This approach, in contrary to the previous one, is a type of analysis in which the projection under investigation can be estimated a-posteriori.

In addition to the above regression model a deformation analysis can be applied in terms either of *Tissot* or of *strain* ellipses, which lead to map-projection characteristics. In this case, a curvilinear deformation analysis can also be carried out directly on a spherical surface, utilising simplified geodetic computations (Livieratos, 2001) in association to geographic coordinates and spherical trigonometry without projecting onto a plane surface avoiding thus, alterations induced due to the projection.

### **The projection-fitting tests**

#### *The image-to-image approach*

Using the raster-wise analysis in the field of the projection-fitting approach two tests were followed. The first (Balletti, 2001) is following the straightforward comparison of  $m'$  with  $m$  counterparts in given projections. The  $m$  model used was a Mercator map of the Mediterranean issued by the Italian Hydrographical Institute with contact parallel at 41.5 degrees North. This map's raster file was digitally transformed using MatLab

utility in a series of eight  $m$  derived files covering a broad spectrum of map projections with different deformational properties (i.e. Apianus, Bonne, Equidistant cylindrical, Equidistant conic, Conformal conic, Mercator normal, Sinusoidal, Werner). All eight maps  $m$  have direct origins at or are by-products of well known historical map-projections mainly from the XVI century. Four nautical old maps representing Mediterranean, from the Correr Museum of Venice map-collection were used for the creation of relevant raster-files  $m'$  (The Correr portolan maps No. 35 from the first half of the XVI century and No. 43 from the XVII century, both by anonymous mapmakers and the portolan maps No. 16 and No. 33 by Bartolomeo Olives and Giorgio Sideri respectively from the second half of the XVI century). These four  $m'$  raster files were fitted optimally, via Helmert's transformation, to the eight  $m$  raster files using 82 to 86 control points distributed along the Mediterranean coast-line. The transformation was carried out using an "image-to-image" transformation within the I/RAS C environment and iso-lines of the residuals were plotted depicting the spatial distribution of the residuals over the Mediterranean. All fittings of nautical maps  $m'$  to cylindrical and pseudo-cylindrical projections  $m$  give rotations of the north axis varying from 8.1 to 10.2 degrees whereas the fittings of  $m'$ s to conic and pseudo-conic projections  $m$  give relevant rotations from -4.3 to 2 degrees. Using the evaluation criterion of the standard error as derived from the image-to-image fitting, the best solution for all tests applied to all four  $m'$  is associated to the choice of the Mercator normal projection as the best fitted model projection  $m$ .

#### *The image-to-map approach*

The second case follows an image-to-map "trial and error" procedure (Boutoura, 2000) due to which, a map  $m$  in a proper projection is searched, in order to fit optimally  $m'$ . For an actual test satisfying this procedure a Mercator map of the Mediterranean was used issued by the US Department of Defence with contact parallel at 40 degrees North. This map was digitised vector-wise and transformed using the Intergraph's Projection Manager utility in order to obtain the relevant a geographical coordinates vector file which was then transformed to  $m$  selected to fit optimally  $m'$  using an "image-to-map" transformation within the I/RAS C environment. The selection was done following a direct "trial and error" procedure within the visual graphics environment. The test involved for  $m'$  map the Giorgio Sideri (1565) nautical map -BNF, National Library of France, No. 4497- and for  $m$  maps, a normal Mercator projection with standard parallel at 36 degrees and an oblique conformal conic projection of Lambert's type degenerated first to a normal equatorial cylinder and then tilted into a cylindrical oblique version with central meridian at 10 degrees East (passing from Pisa) in 45 degrees azimuth. This second oblique conformal projection was derived as the best possible from the "trial and error" procedure of on-line testing. This oblique version complies to certain hypotheses defended in the existing literature, namely to the obliquity of cylindrical and conic projections which might be used -among other possible projections- by mapmakers or map-copyists. The choice of this oblique conformal projection gave satisfactory results from the very first approximation of the fitting process, both in configuration and in

numbers, since the final fitted old nautical maps are optimally approximated, keeping at the same time almost unaltered their initial orientation.

The fit to the normal Mercator projection gave a 9.9 degrees rotation of the north axis and the fit to the oblique projection a  $-1.7$  degrees counterpart. According to the standard error criterion as derived from the image-to-map fitting, the oblique conformal cylindrical projection defined above, gave 20% better results compared to the normal Mercator projection.

Concerning the Aegean Sea, two *m'* nautical maps were used, one by Pietro Vesconte (1313) and the other one by Antonio da Millo (1750) both from the Correr Museum of Venice -No. XX and No. XX respectively-. For the fitting, the above oblique conformal projection was used as the *m* model. It is impressive the consistency of the fitting of Vesconte's Aegean map within Sideri's map of the Mediterranean in relation to the oblique conformal projection defined here (Boutoura, 2000b). In this context it is interesting to stress also the coincidence of the results obtained for the Antonio da Millo map with respect to those from another relevant study dedicated to the same map (Boutoura, 2000a). Both maps depicting the Aegean Sea the rotation of the north axis, after the fitting, is 1 degree for the Millo map and 0.2 degrees for the Vesconte map.

### **Concluding remarks**

This paper presented some new digital methods, which can be used in studying the projective properties of historic maps, with specific interest to relevant representations of the Mediterranean and the Aegean seas. These methods are based mainly on the raster-wise image processing of digital map files. The digital files of old maps are compared within a unique analytical and graphical environment with relevant digital files of actual maps using image-to-image or image-to-map transformations. These processes could handle and treat massive sets of map files allowing the choice of alternative ways of modelling and testing allowing on-line visual decision making in accepting or rejecting theories and options.

Actual tests with old nautical maps representing the Mediterranean and the Aegean Sea areas converge to some general conclusions related to the projective properties of old nautical maps concerning e.g. conformality, rotations and areas with systematic deformation patterns like the North part of the Aegean Sea.

Another conclusion is that the use of the numerical evaluation criteria, like e.g. the standard error of the fit should be considered in combination with the spatial pattern of the distribution of the residuals. The "trial and error" approach, lead to interesting solutions requiring on the other hand systematic tests and intuition in selecting variants of known projections which fit best the old maps.

It is evident that the types of analyses presented here contribute further but do not solve in a unique way the still open problem of the projective nature and origin of the nautical maps. It gives a new impulse and insight in this field of studies, showing the great potentiality of modern digital tools and common processing environments with visualisation, which indeed facilitate the testing of alternative hypotheses and theories.

Further research should be done in evaluating and then integrating the methods combining the best fitting with deformation analysis testing systematically a great number of old maps in order to construct consistent interpretation models on the projective properties of old maps.

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