BOOLEAN ALGEBRAIC ORGANIZATION OF CARTOGRAPHIC SYMBOL SYSTEM AND ITS APPLICATION IN THE COMPUTER-AIDED CARTOGRAPHY

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Map image system with the point as the basic element belonging to Boolean algebraic one has been proved by the author.[1] The cartographic symbol system with symbol as the basic element belonging to Boolean algebraic one is further proved by the authors in this poper .

1 Boolean algebraic definition and its sufficient conditions[2] ; the proof of cartographic symbol system belonging to Boolean algebraic one

1.1 Partially orderod set

Definition 1 The $(L,\leqslant)$ is called the grant ordered set, if relation on L suffices three conditions as follows:

1) Reflexive: $a\leqslant a$;
2) Anti-symmetric: if $a\leqslant b$ and $b\leqslant a$,then $a=b;$
3) Transitive: if $a\leqslant b$ and $b\leqslant c$,then $a\leqslant c$.

where $a ,b ,c \in L$, $(L, \leqslant)$ is called the partially ordered set.

1.2 The partially ordered relation in the cartographic symbol system

1.2.1 The discussion field and power set of cartographic symbol system

Let X be a certain map ,It is the set of cartographic symbols which compose this map .we denote

$$T(X)=\{A;A\subset X\} \tag{1}$$

Here $T(X)$ is a power set of X. It is agreed that $\phi, X\in T(X)$,if the number of elements is n in X, then in $T(X)$ the number of elements are 2^n.

It is obvious that the set composed of individual cartographic symbols and different cartographic symbols are the subset of $T(X)$.

1.2.2 The partially ordered relation on the power set is the inclusive relation

If $T(X)$ is the power set of X, let $P= T(X)$,then the inclusive relation" $\subset$" is the partially ordered relation on P , namely It suffices:

1) Reflexive: $A\subset A$;
2) Anti-symmetric: if $A\subset B$ and $B\subset A$,then $A=B$;
3) Tramsitive: if $A\subset B$,$B\subset C$,then $A\subset C$.

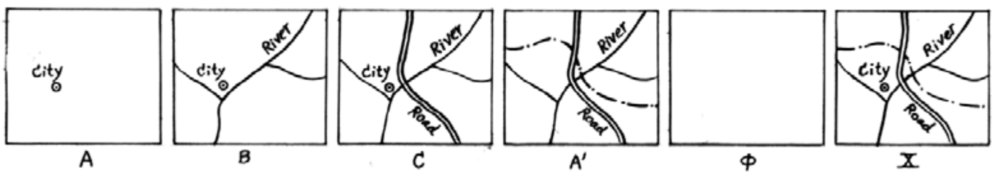

Fig.1 Diagram of Cartographic Symbol System

1.2.3 The inclusive relation in cartographic symbol system

There exists the inclusive relation in the cartographic symbol system as follows (Fig 1):

$\Phi \subset A \subset B \subset C \subset X$

1.3 Lattice

Definition 2 The partially ordered set $(L, \leqslant)$ is called the lattice ,if for any $a, b \in L$ there are $a \vee b$ and $a \wedge b \in L$.

In Fig. 1 ,It is obvious that $A \vee \Phi = A$, $A \vee B = B$, $B \vee C = C$, $C \vee X = X$; $A \wedge \Phi = \Phi$ $A \wedge B = A$, $B \wedge C = B$, $C \wedge X = C$. Because of the cartographic symbol system suffices the definition of lattice, so it is a lattice.

1.4 Distributive lattice

Definition 3 If $(L, \leqslant)$ suffices the distributive law ,namely $\forall a, b \in L$,them we have

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

then $(L, \leqslant)$ is called distributive lattice.

In Fig. 1, $A, B, C \in P$,$(P, \subset)$ suffices the distributive law ,namely

$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C) = B$, $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C) = A$

so $(P, \subset)$ is the distributive lattice.

1.5 Boolean algebra

Definition 4 Let $(L, \leqslant)$ be the lattice ,it has the smallest element 0 and largest element 1 and $\forall a \in L$,if there exists $a' \in L$,which makes $a \wedge a' = 0$, $a \vee a' = 1$,then L is called the complemented lattice, a' is the complement of a .The complemented distributive lattice is called Boolean algebra .

In Fig. 1 , $A \in P$, $A' = (1-A) \in P$, and $A \wedge A' = \Phi$, $A \vee A' = X$,so A' is a complement of A , Φ is the smallest element and X is the largest element.

1.6 The necessary and sufficient conditions for Boolean algebra

Theorem 1. The necessary and sufficient conditions for Boolean algebra is that there exist two operations of $\vee$ and $\wedge$,and suffice:

1) $a \wedge b = b \wedge a$,$a \vee b = b \vee a$ (commutative law)
2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ (distributive law)
3) there exist 0 and 1 ,which can make $a \vee 0 = a$,$a \wedge 1 = a$ (unitary element exists)
4) $\forall a \in L$ and $a' \in L$ exists ,which can make $a \wedge a' = 0$,$a \vee a' = 1$

1.7 The proof of cartographic symbol system belonging to Boolean algebraic one

Proof: Let the different symbols in theorem 1 respectively corresponds to the symbol in Fig .1,namely: a-A ,b-B ,c-C ,a'-A' ,0- Φ,1-X .For two operations of $\vee$ and $\wedge$,there exist

1) $A \wedge B = B \wedge A = A$, $A \vee B = B \vee A = B$ (commutative law)
2) $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C) = B$, $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C) = A$ (distributive law)
3) $A \vee \Phi = A$, $A \wedge X = A$

where Φ and X are unit elements in the cartographic symbol system respectively (*i,e.*,there exist unit elements)

4) $\forall A \in P$, $A' \in P$, $A \wedge A' = \Phi$, $A \vee A' = X$, (relatively complement law)

Due to the cartographic symbol system $(P, \subset)$ satisfies necessary and sufficient conditions in Boolean algebra ,so it belongs to the Boolean algebric system.[3]

2 The meaning and application of Boolean algebra in computer-aided cartography

2.1 The Boolean algebraic system with point as basic element is the theoretical basis of cartographic symbol design in computer-aided cartography

The map is a model of figure-symbol for the objective reality .The cartographic symbols are "modelling graphic language", it indicates the abstract concept with visual figure and has the specific property of visualization and spacial presentation .The cartographic symbols mainly includes three types of visual image ,namely the graphic symbol ,colour symbol and language symbol.The cartographic symbolic external character are produced through the different composition of visual variables: the form ,size ,direction ,lightness ,colour and place ,in a word ,we can get it through the place and colour variation of the point .The Boolean algebraic structure of map image system with point as the basic elememt gives the theoretical basis and mathematical tools for computer-aided symbol design.Therefore, it creates favorable conditions for construction,rich and update of database .This no doubt that , it has important meaning for raising map quality on the whole and expanding its service field .

2.2 Boolean algebraic structure of cartographic symbol system is the theoretical basis and mathematical tools for construction of map contents on mnlti-levels

2.2.1 The level principle and its formation mode in the cartographic symbol system

Definition 5 Let $\{X_i\}_{i\in I}$ be a family of subset of known set X, *i.e.*,$\forall i \in I$, $X_i \subset X$,if conditions is sufficed:

1) $X = \cup_{i\in I} X_i$;
2) $\forall i \in I$, $X_i \neq \phi$;
3) $\forall (i, i') \in I \times I', i \neq i' \Rightarrow X_i \cap X_{i'} = \phi$

then the set family $\{X_i\}_{i\in I}$ is called a classification of set X .Every X_i is called a category of the classification $\{X_i\}_{i\in I}$.

Theorem 2 Equivalent relation "$\sim$" on set X decides a classification of X ,and causes element x and x' of X belonging to one category $\Leftrightarrow x \sim x'$.On the contrary ,every classification $\{X_i\}_{i\in I}$ of set X decides an equivalent relation "$\sim$" on X .Its definition is as follows:

$x \sim x' \Leftrightarrow x$ and x' belonging to the same category X_i .

In cartographic practice for $x \in X$ we usually define three equivalent relations and thus three classifications can be obtained:

1) The symbol character i is defined as equivalent relation , *i.e.*

$$X = \cup_{i\in I} X_i \tag{3}$$

The classification of cartographic symbol according to the residential area ,river system *etc* .belongs to this one .

2) The symbol colour j is defined as equivalent relation , *i.e.*,

$$X = \cup_{j\in J} X_j \tag{4}$$

The classification of cartographic symbol according to the colour: yellow, red,blue,etc. belongs to this one.

3) The level t of thick and thin or ruling scale is defined as equivalent relation , *i.e.*,

$$X = \cup_{t\in T} X_t \tag{5}$$

The classification of cartographic symbol according to ruling scale10% ,15% ,20% ,*etc* belongs to this one .

The above three classifications construct the relation including relation and level in cartographic symbol system:

$$\left.\begin{array}{l} \phi \subset x \subset X_i \subset X \\ \phi \subset x \subset X_j \subset X \\ \phi \subset x \subset X_t \subset X \end{array}\right\} \qquad (6)$$

2.2.2 The application of level principle in cartography

In conventional cartography the level shows the form of colour dividing ,platemaking and overprint *etc* .In computer-aided cartography the construction of map may begin from the level of possessed condition and with it as basic"letter"[4] .The practice of computer-aided cartography shows that the theory of the construction of map contents by multi-level algebraic operation is correct and the mothod is advanced .

3 Conclusion

The proof of cartographic symbol system belonging to Boolean algebraic one and expression of level principle of the system furnish the theoretical basis and mathematical tool for computer-aided map compilation,at the same time ,it provides theoritical support for the practice of multi-level construction of map contents in computer-aided mapping ,too. Boolean algebraic structure of cartographic symbol system will provide wide prospects for the exploitation and application of the software in computer-aided cartography.

Reference Literatures

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