TEMPORAL RELATIONSHIPS FOR GEO-SPATIAL OBJECTS
HAVING SPATIO-TEMPORAL ATTRIBUTES

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Abstract
The aim of this paper is to introduce the taxonomy of the spatio-temporal relationships for geo-spatial objects. Feature defined as abstraction of real world phenomena has attributes in 3-D space, 1-D time or 4-D space-time. These attributes are called geo-spatial objects in this paper. They are classified into geometry and topology. Geo-spatial objects may occur, move, deform, destroy and succeed to other objects. This paper sorts these spatio-temporal phenomena.

Introduction
There have been several researches about spatio-temporal model for geo-spatial objects [e.g. Erwig, et al., 1999]. They mainly aim to describe movement as displacement of a spatial object, while they did not discuss about succession. Succession is defined as replacement of preceding objects by succeeding objects. This type of phenomenon causes the transition of topology. For example, the phenomenon 楓森林 is changed and divided to a pasture and a road” means that a forest closes its temporal limit at the moment of division, then a pasture and a road of which beginning limits occur.

This paper aims to expand the dimension of spatio-temporal attributes proposed in [Ota, 1999] to 3D spatial and 1D temporal, and to discuss about the spatio-temporal relationships among dynamic geo-spatial objects. For this purpose, this paper, at first, discusses about the definition of feature and related terms. Next, characteristics of space, time and space-time are discussed as the basis of the description of temporal relationships among spatio-temporal objects. Then, topological characteristics of geo-spatial object抯 motion are introduced. Finally, conclusions and discussion points for further considerations are mentioned.

Feature, Object, Type, and Class
This section discusses about feature and related concepts. In conventional cartography, feature has been defined as an object on a map. It is, for example, a symbol of building, line of road or river. However topography as the base on which features is drawn is not categorised in a feature. Recently, feature is defined as 撛group of spatial elements which together represent a real-world entity” [McDonnell, Kemp, 1995]. Spatial elements such as point, line, area and complex of them are figures. In this definition, topography is also a feature as it is a real-world entity. In Spatial Data Transfer Standard (SDTS) [U.S. Department of Commerce, 1992], a feature is defined as the combination of the phenomenon and its representation. This definition clarifies two aspects of real world entity, phenomenon as type and representation as spatial object. These definitions still treat a feature as the graphic representation of real world phenomena. Since GIS community started to apply object-oriented design technique in late 1980s, the concept of a feature was redefined. For example, Worboys (1994) pointed out 撛the key notation is object = state + functionality”. He also mentioned that objects with similar behaviours are organized into types, while classes are groupings of objects with corresponding data structures and methods. Today, in the standards on geographic information like ISO/TC211, definitions of object, type and class are derived from related IT standardisation efforts, for example UML standardised by OMG. The definitions of object, class and
type are as follows.

**Object:** An entity with a well-defined boundary and identity that encapsulates state and behaviour. Attributes and relationships represent state; operations, methods, and state machines represent behaviour. An object is an instance of a class. [OMG, 1997].

Where, state machine is defined as 揦behaviour that specifies the sequences of states that an object or an interaction goes through during its life in response to events, together with its responses and actions”

**Class:** A description of a set of objects that share the same attributes, operations, methods, relationships, and semantics [OMG, 1997].

**Type:** A stereotype of class that is used to specify a domain of instances (objects) together with the operations applicable to the objects [OMG, 1997].

While, feature is a special class defined by the inherent knowledge of Geographic Information Science. The definition given by ISO/TC211 is,

**Feature:** abstraction of real world phenomena. A feature attribute has a name, a data type, and a domain of attribute value associated to it. A feature attribute for a feature instance also has an attribute value taken from the domain.

In this paper, if a feature relates to the position of the earth, it is called geo-spatial feature. And its instance is called geo-spatial object. As long as geo-spatial feature is abstraction of phenomena of the real world, it can be observed as a substance in space, time or space-time. Therefore, before to discuss about spatio-temporal relationships among features, we need to discuss about characteristics of space, time and space-time.

**Space and spatial objects**
According to Gatrell (1991), space is defined as a relation on a set of objects. Space where we live in can be mapped on to 3-D Euclidian space. Where, a relation is metric and measured by distance between two objects as: \( d(i, j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \). In Euclidian space, distance between two objects is symmetric: \( d(i, j) = d(j, i) \); they obey so-called triangle inequality: \( d(i, k) \leq d(i, j) + d(j, k) \). As space is 3-D, geometric primitives (non de-composable geometric object) in space are primally classified to point, curve, ring, surface, shell and solid (See Table 1). Point is a position in space. Curve is a sequence of points interpolated by a certain method. Primitive interpolation method may be liner. Ring is a closed curve. Surface is a figure of which boundary is a set of closed curves. It may have halls. Shell is a closed surface (e.g. a bubble) in 3-D space. Solid is a closed figure of which boundary is a composition of surfaces. It may have shells inside. In this paper, geometric objects are simple and it does not have self-intersection.

<table>
<thead>
<tr>
<th>Primitives</th>
<th>Definitions</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
<td>( \text{Point} = (x, y, z) )</td>
<td></td>
</tr>
<tr>
<td>Curve</td>
<td>( \text{Curve} = {\text{point}} )</td>
<td>Number of points in a sequence is greater than 1.</td>
</tr>
</tbody>
</table>
Ring

Ring = curve

Curvature, \( p_0 \) and curve, \( p_{n-1} \) in a curve are equal, \( n > 2 \). Rotating direction is anti-clockwise.

Surface

Surface = (outer_ring, \{inner_ring\})

Each ring is disjoint from others. The left side of ring is outside of a surface, if it lies on the 2D space.

Shell

Shell = \{surface\}

A boundary of a surface must coincide with a boundary of an adjacent surface. Normal vector of each surface is from inner to outer.

Solid

Solid = (outer_shell, \{inner_shell\})

Each shell is disjoint from others.

(a, b, ..., k): a tuple of a, b, ..., k,

\{a, b, ..., k\}: a set of a, b, ..., k,

\(<a, b, …, k>: an ordered set of a, b, …, k

(a): a tuple of \( a_i \), \( i=0..n-1 \)

\{a\}: a set of \( a_i \), \( i=0..n-1 \)

\(<a>: an ordered set of \( a_i \), \( i=0..n-1 \),

The spatial model mentioned above is merely an example, as Euclidean space is one type of metric spaces. For example the distance in a road network is not same as the distance in Euclidean space.

Topology is also considered in space. It is spatial connectivity relation among objects invariant when their relative position change continuously or elastically. Connectivity is boundary and co-boundary relationships among adjoining geometries. If you think about the actual road network, it is topological, because at one-way, distance is asymmetric: \( d(i, j) \) can be measured but \( d(j, i) \) cannot. Topological primitives in space are node, edge, loop, face, case and block (See table 2). They are explained in Table 2.

<table>
<thead>
<tr>
<th>Primitives</th>
<th>Boundarys</th>
<th>Co-boundarys</th>
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</thead>
<tbody>
<tr>
<td>Node</td>
<td>-</td>
<td>{leaving_edge}, {reaching_edge}</td>
<td>Rotation direction of left_face is same as the edge, while the direction of right_face is opposite to the edge.</td>
</tr>
<tr>
<td>Edge</td>
<td>start_node, end_node</td>
<td>{left_face}, {right_face}</td>
<td>Loop is the closed sequence of edges; the start edge connects to the end edge. Inner_face is at the left side of the loop on a 2-D manifold.</td>
</tr>
<tr>
<td>Loop</td>
<td>-</td>
<td>inner_face, outer_face</td>
<td>Outer and inner loops lie on the same 2-D manifold. Front_block is a block at the normal direction to a face. While, back_block lies on the back side.</td>
</tr>
<tr>
<td>Spatial face</td>
<td>outer_loop, {inner_loop}</td>
<td>front_block, back_block</td>
<td>A case is a composite set of faces. It is a boundary of a block.</td>
</tr>
<tr>
<td>Spatial case</td>
<td>-</td>
<td>front_block, back_block</td>
<td></td>
</tr>
</tbody>
</table>
Spatial topology represents the boundary and co-boundary relationships among spatial objects. Node is a boundary of edges. Node has no boundary. Edge is a component of a boundary of a face. Loop is a boundary of a face; it is a composition of edges. Face is a component of a boundary of a block. Case is a boundary of a block; it is a composition of faces. Block is a 3-D topological object; its boundary is a set of cases.

**Time and temporal objects**

Time is one-dimensional metric space. Distance between two objects is defined as $d(i, j) = \sqrt{(t_i - t_j)^2} = |t_i - t_j|$; distance is symmetric: $d(i, j) = d(j, i)$; they obey the equality: $d(i, k) = d(i, j) + d(j, k)$, this is the special case of triangle inequality. But almost all phenomena in time are asymmetric between the past and the future. Cause does not happen after effect. It means that the beginning time of a phenomenon is always earlier than its end time. Time does not have a direction, while phenomena occurs toward one direction in the usual world.

There are two types of geometric primitive (See Table 3). Instant is a temporal position, while period is an interval of time. Pair of instant explaining its start and end represents a period.

<table>
<thead>
<tr>
<th>Table 3. 1-D temporal geometry</th>
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</thead>
<tbody>
<tr>
<td><strong>Primitive</strong></td>
</tr>
<tr>
<td>Instant</td>
</tr>
<tr>
<td>Period</td>
</tr>
</tbody>
</table>

Topology in time is connectivity between preceding and succeeding objects at their end-to-begin. A phenomenon happened when preceding objects is replaced by succeeding objects is called succession. The connectivity may be many-to-many relationship. Related objects may not have any spatial relationships, because time is independent from space in the usual real world. For example, when a person having two parcels purchase another parcel at a different place, his/her ownership changes from two to three. It can be considered as that his/her parcels succeeded from two to three by an event "purchase". Geometric time is mapped on to a line. However, Topological time is mapped on to 1-D network.

<table>
<thead>
<tr>
<th>Table 4. 1-D temporal topology</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primitives</strong></td>
</tr>
<tr>
<td>Node</td>
</tr>
<tr>
<td>Edge</td>
</tr>
</tbody>
</table>

Topological primitives in time are a node and an edge. Edge has to have a direction from begin to end, as phenomena occur toward one direction. Therefore, relationships among topological primitives construct an acyclic directed graph, which is a special type of 1-D network. Parcels in the above example are abstracted to edges; the event "purchase" is abstracted to a node. The lineage of his/her parcels constructs an acyclic directed graph.

Acyclic directed graph grows by growing of edges and successions. There are 4 types of successions [Ota, 1999], substitution, division, fusion, and their complex. Substitution is that one object occurs from one object. Division is many from one. Fusion is one from many. And complex is many from many. Creation and destroy are also defined as null from one and one from null. Therefore, temporal
Topological phenomena are sorted by three concepts: creation, succession, and destroy.

![Classification of Temporal topological phenomena](image)

**Space-time and spatio-temporal objects**

Every phenomenon in the real world exists in both space and time. Because, an existence does not have shape and relationship without space, and its change cannot be measured without time. In the real world, space is not independent from time, because spatial objects change in time. Typical example is a movement of a point. It is represented by a function at a certain time \( t \); \( p = f(t) \) or \( f: \mathbb{R} \rightarrow \mathbb{R}^3 \). A function \( f \) can be obtained empirically by observations of the sequence of position \( (x, y, z, t) \). Position at a moment is called a point. Even if \( f \) is not defined, the position can be estimated by interpolation. Therefore, a sequence of points and the choice of interpolation method are the essential requirements to describe a point motion. The case that a position does not change during its lifetime is also a special case of a motion. Table 5 defines spatio-temporal geometric primitives.

**Table 5. spatio-temporal geometry**

<table>
<thead>
<tr>
<th>Primitives</th>
<th>Definitions</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
<td>Point = (x, y, z, t)</td>
<td>Point represents a position at a moment.</td>
</tr>
<tr>
<td>Curve</td>
<td>Curve = &lt;point&gt;</td>
<td>Curve is a motion of a point constructed by a sequence of its positions observed in order. curve.point(<em>i) ( \leq ) curve.point(</em>{i+1}), ( t ).</td>
</tr>
<tr>
<td>Ring</td>
<td>Ring = (curve)</td>
<td>Ring is a boundary of a surface, which is constructed by a set of curves lying on 2-D space, connecting with a different curve at a boundary. The example shows that 1. Two points separate to four. 2. Two of them join to one, and then it disappears. 3. Lest two move until they join to one, then it disappears. These points are parts of one feature.</td>
</tr>
<tr>
<td>Surface</td>
<td>Surface = (outer_ring, {inner_ring})</td>
<td>Surface is a motion of a curve sometimes with breaks, which is constructed by an outer ring and inner rings lying on 2-D space. The example shows that, 1. Two points appear and they grow to two lines. 2. Two lines join to one.</td>
</tr>
</tbody>
</table>
3. One line separates to two.
4. Two lines join again and it reduces to a point.
5. Point disappears.
These lines are parts of one feature.

Shell

Shell = (surface)

Shell is a boundary of a solid, which is constructed by a set of surfaces. Each surface must connect with a different surface at a boundary. The example shows that a boundary of a square reduces its size as time passes and finally it disappears.

Solid

Solid = (outer_shell, {inner_shell})

A motion of a surface sometimes with halls. It is constructed by an outer shell and may have more than one bubble. The example shows a motion of a surface and its hall appearing during the motion. Finally, one square disappears at the end time.

Hyper-shell

HyperShell = (solid)

Hyper shell is a boundary of a hyper solid, which is constructed by a set of solids. Each solid must connect with a different solid at a boundary surface.

Hyper-solid

HyperSolid = (outer_hyperShell, {inner_hyperShell})

A motion of a solid sometimes with bubbles.

A motion is a spatio-temporal object changing its shape as time passes. The temporal section of a spatio-temporal object (a spatial geometry sliced at an arbitrary moment during an existence period of an object) may consist of more than one part, however they have same identity preserved by the fact that they belong in one spatio-temporal object. Every spatial object lying on a temporal section synchronises.

The discussion above is endorsed by Newtonian discipline. GISs are usually applied to explain the phenomena related to the earth. Origin of axes is the gravity centre of the earth, and their temporal reference system is UTC or local time transferable to UTC.

Topology in motion

Motions of spatio-temporal objects are abstracted topologically as static directed topological objects (See Table 6). As mentioned already, successions deliver an acyclic directed graph in time. A motion of a spatial primitive has one to one relationship to a temporal edge. Moreover, motions may cause other successive motions. Through a motion and an event is replaced by a directed edge and a node, we can define a spatio-temporal complex describing a lineage of motions.

Table 6. Spatio-temporal topology

<table>
<thead>
<tr>
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<th>Boundarys</th>
<th>Co-boundarys</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
<td>-</td>
<td>({leaving_edge}, {reaching_edge})</td>
<td>Node is 0-D topological boundary of edges. A leaving edge has a node at beginning. While reaching edge has it at end.</td>
</tr>
<tr>
<td>Edge</td>
<td>(beginning_node, end_node)</td>
<td>{face}</td>
<td>Edge is 1-D topological object. A beginning node is older than or simultaneous with an end node.</td>
</tr>
<tr>
<td>Loop</td>
<td>-</td>
<td>({outer_face}, inner_face)</td>
<td>Loop is 1-D topological boundary of one inner face. Loop is a composition</td>
</tr>
</tbody>
</table>
of edges. Edges lie on the 2-D manifold. As every edge has a direction from older to newer, loop is constructed by edges having a sign to make them a cyclic sequence.

<p>| | | | |</p>
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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Face</td>
<td>(outer_loop, {inner_loop})</td>
<td>{block}</td>
<td>Face is 2-D topological object. It is a boundary of two blocks on the 2-D manifold.</td>
</tr>
<tr>
<td>Case</td>
<td>-</td>
<td>({outer_block},</td>
<td>Case is 2-D topological boundary of one inner block. It is constructed by a composition of faces, which boundaries are corresponding with adjacent faces.</td>
</tr>
<tr>
<td>Block</td>
<td>(outer_case, {inner_case})</td>
<td>{hyperBlock}</td>
<td>Block is 3-D topological object. It is a boundary of hyper blocks.</td>
</tr>
<tr>
<td>Hyper case</td>
<td>-</td>
<td>({outer_hyperBlock}, {inner_hyperBlock})</td>
<td>Hyper case is 3-D topological boundary of one inner hyper block. It is constructed by blocks, which boundaries are corresponding with adjacent blocks.</td>
</tr>
<tr>
<td>Hyper block</td>
<td>(outer_hyperCase, {inner_hyperCase})</td>
<td>-</td>
<td>Hyper block is 4-D topological object.</td>
</tr>
</tbody>
</table>

**Characteristics of succession among spatio-temporal objects**

(1) The dimension of a boundary at a moment of division and fusion

The dimension of boundaries between dividing or fusing primitives at a moment of succession is less than or equal to the dimension of the primitives. For example, when surfaces divide from a preceding surface, the boundary at a moment of dividing may be a point, a curve, or a surface.

![Figure 2. Dimension of a boundary at a moment of division](image)

(2) Branching and mergence

There is a case that a moving object generates a set of new objects. While, a set of moving objects may merge into one movement. These phenomena occur when there is a movement not to make a limit boundary at a moment of succession. In this paper these phenomena are called branching and mergence.

![Figure 3. Branching and mergence](image)
(3) Pseudo-division and pseudo-fusion
There are cases that none of moving objects generates new objects. In this case, division does not create other movements, and fusion does not amalgamate other objects. We name these types of succession as pseudo-division and pseudo-fusion. These cases are nothing but a spatio-temporal block.

![Figure 4. Pseudo successions](image)

Conclusion
This paper discussed about the definition of a feature at first. A feature has 3-D spatial, 1-D temporal or 4-D spatio-temporal attribute. These attributes are selected by the purpose of abstraction from the real world. A moving or deforming object is an existence in space-time. This paper introduced the spatio-temporal geometry and topology models. Then, the taxonomy of spatio-temporal events between preceding and succeeding features was introduced.

There are many other possibilities to describe spatio-temporal phenomena, for example, reincarnation could be a temporal continuous and spatio-temporal discontinuous phenomenon. We should check the possibilities of these phenomena in space-time.

This paper did not discuss about influences against adjacent features caused by a motion or a deformation of a feature. These issues will be considered in the future.

Bibliography