

## MODELING DEM UNCERTAINTY IN GEOMORPHOMETRIC APPLICATIONS WITH MONTE CARLO-SIMULATION

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### Abstract

The paper presents studies how digital elevation model (DEM) uncertainty affects common terrain analyses, such as slope and aspect calculations as well as watershed delineation. The work is based on Monte Carlo-simulation, in which the vertical DEM error was modeled by spatially autocorrelated isotropic homogeneous Gaussian random fields (GRFs). A convolution technique for adjusting the spatial autocorrelation of the GRF is also introduced.

The study showed, that increase in vertical DEM error increased the variance of the DEM derivatives. The effect of spatial autocorrelation was dependent on the application. For local surface derivatives, such as slope and aspect, maximum variance was reached when the range of the predefined spatial autocorrelation function corresponded the size of the derivative's calculation window. The uncertainty in watershed delineation increased, while the range of spatial autocorrelation increased within the predefined limits. Monte Carlo-simulation appeared to be appropriate error propagation analysis method for investigating both local and regional surface derivatives.

### Introduction

DEM geomorphometry and terrain analysis are used extensively in many environmental disciplines, such as physical geography and hydrology (e.g. Giles & Franklin 1998, Moore 1996). Among these sciences, the DEMs are often used as error-free models of reality, even though the existence of elevation uncertainty and even gross errors are widely recognized in surveying sciences and geoinformatics (e.g. Heuvelink 1998). The quality of the elevation model can be evaluated with field survey or photogrammetric measurements, but spatially and morphologically representative sampling has appeared to be very difficult and expensive task. In addition, global error statistics, such as RMSE, give a general estimate of the DEM quality, but still the DEM user does not know how the errors might affect specific analysis results.

The effect of DEM uncertainty in terrain analysis is possible to investigate with analytical probabilistic error propagation techniques or with Monte Carlo –simulation approach. As a numerical computation method, Monte Carlo –simulation bypasses analytical error propagation approach. This is particularly important in regional modeling tasks, in which the analysis result is a function of far reaching and unpredictable spatial interactions.

The study presents 16 scenarios, which represent how DEM uncertainty might affect common surface derivatives, such as slope and aspect, as well as watershed delineation. The different scenarios were calculated by setting the predefined parameters for the standard deviation and spatial autocorrelation of the vertical DEM error. The research was carried out with the DEM of Ruissalo island, located 5 km West from the center of Turku city in South-Western Finland. The size of the DEM was 652×440 height points

having spacing of 10 meters and it was interpolated with the LIDIC-method (Oksanen & Jaakkola 2000) from the contours of the large-scale town planning maps produced by the city of Turku.

### DEM-based slope, aspect and watersheds

When considering the terrain elevation as a continuous function of location  $z=f(x,y)$ , the first derivative of the surface is a gradient vector formed of slope and aspect components. Slope is the rate of elevation change in steepest descent direction and it is of great significance in environmental modeling, because it controls the gravitation driven material flow on a surface. Aspect is the direction of the steepest slope. The major importance of aspect is its influence on spatial pattern of insolation, but it also indicates the bedrock fracture structure and directions of ancient glacial flows in the landscapes of streamlined surficial deposits. Calculation of slope and aspect was based on the first partial derivatives of  $z$ , which were approximated in the study by the method proposed by Horn (1981).

The watersheds were derived from the flow directions calculated by the D8-algorithm (Jenson and Dominique 1988). In the study the interest was only on watersheds, and thus the deficiencies of D8-algorithm (e.g. Costa-Cabral & Burges 1994, Tarboton 1997) could be ignored. The primary watersheds were defined as the borders of the aggregated drainage basins, in which the aggregation was based on the contribution of flow to the sea area North, South-East and South-West of the Ruissalo island.

### Analyzing the DEM error propagation

For error propagation analysis purposes the DEM is assumed to be one realization of a stochastic surface of random variables, which are possible to describe in each elevation point location by their expectation value associated with parameter of density function, such as standard deviation in normal distribution, and by their spatial relationships. The idea of Monte Carlo-simulation in DEM error propagation is to compute the terrain parameters repeatedly from the DEM realizations, which can be considered as a samples of stochastic elevation model. In practice, the DEM realization is possible to generate by summing the DEM and the GRF described by chosen parameters. To simulate the random DEM error, several assumptions have to be made: 1) the DEM uncertainty is independent of external factors, such as topography or vegetation cover, 2) the DEM uncertainty is spatially autocorrelated, and 3) the DEM errors have normal distribution.

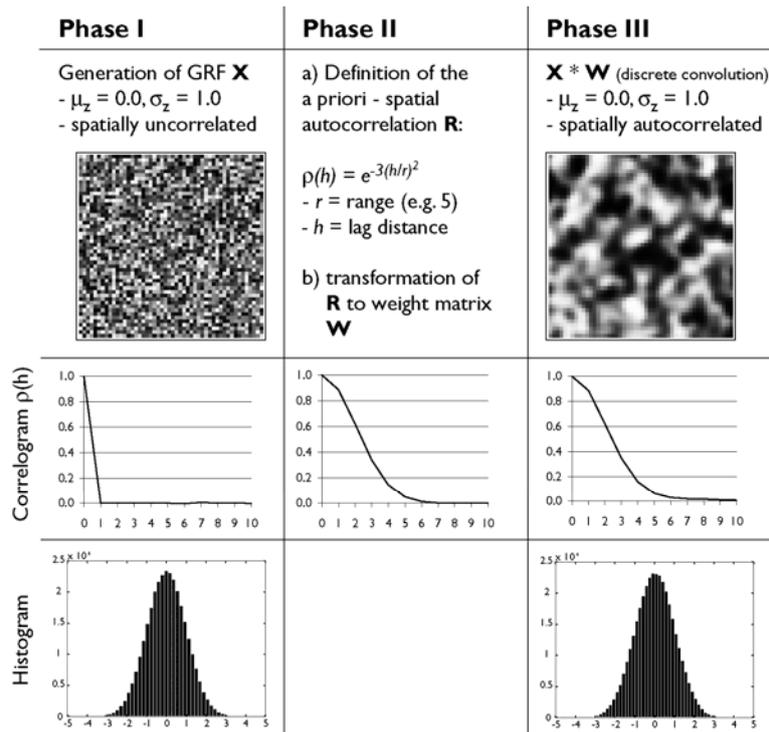
In the study 16 scenarios of DEM error propagation were generated to see the effect of GRF parameters to calculation of surface derivatives. Monte Carlo –simulations were done with 1000 simulation runs, which was a compromise to get reasonably reliable simulation results with moderate computation load. The GRF was described by two parameters: the standard deviation of the vertical DEM error ( $\sigma_z$ ), and the range ( $r$ ) of the predefined a priori-spatial autocorrelation function (Figure 1, Phase IIa). Since the contour interval of the source data was 1.00 m, the values of  $\sigma_z$  were selected as 0.25 m, 0.50 m, 1.00 m, and 2.00 m. Similarly, the spatial autocorrelation ranges were set to 10 m, 20 m, 50 m, and 100 m (or 1-, 2-, 5-, and 10-times the distance between the DEM elevation points).

The generation of spatially autocorrelated GRFs was based on the discrete convolution technique, in which the spatially uncorrelated GRF denoted by matrix  $\mathbf{X}$   $N[0,1]$  (Figure 1, Phase I) was convoluted with weight matrix  $\mathbf{W}$  to gain the spatially autocorrelated GRF  $\mathbf{X}'$  (Figure 1, Phase III) having spatial autocorrelation defined by the a priori correlation matrix  $\mathbf{R}$  (Figure 1, Phase IIa). The  $\mathbf{W}$  was possible to calculate by <sup>1)</sup> squaring the elements of  $\mathbf{R}$ , <sup>2)</sup> taking a matrix square root of this matrix and finally <sup>3)</sup> scaling the elements (Figure 1, Phase IIb).

$$\mathbf{X}' = \mathbf{W} * \mathbf{X} \quad (1)$$

$$\text{Correlogram}(\mathbf{X}') = \mathbf{R} \quad (2)$$

The convolution technique overcame some of the defects appeared in the previously published methods for creating spatially autocorrelated random fields. The limited autocorrelation range used in e.g. Moran's  $I$  or Geary's  $c$  (Goodchild 1986, implementation e.g. Fisher 1991) was resolved by using the Gaussian correlation function (Figure 1, Phase IIa) as the a priori-spatial autocorrelation function. Also the experimental pin-pointing of the weight matrix expressed by Ehlschlaeger 1998 was possible to be bypassed. The explicit definition of the spatial autocorrelation function range gave better intuitive understanding, compared to for example the spatial autoregressive techniques (e.g. Hunter & Goodchild 1997, Heuvelink 1998), for the role of spatial autocorrelation in vertical DEM error.

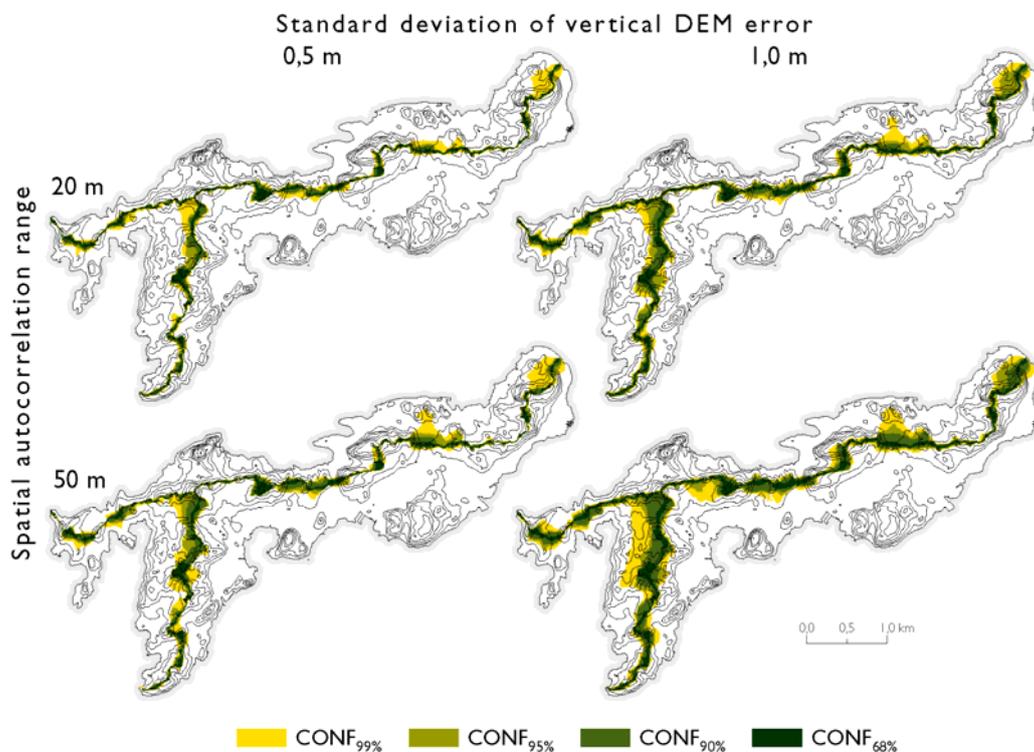


**Figure 1.** Outline of the procedure used for generating spatially autocorrelated Gaussian random fields. The size of the GRF examples above are 500 m  $\times$  500 m (10 m cell size). Correlograms and histograms represent the whole GRFs used in the study having size of 6520 m  $\times$  4400 m.

## Results

The 2  $\times$  2 subset of all 16 scenarios of probabilistic drainage basin delineations is shown in Figure 2. It appeared, that when the spatial autocorrelation range and standard deviation of the vertical DEM error increased, also the uncertainty of the drainage basin delineation increased. In general the borders appeared to have three distinct characteristics. *Crisp borders* were shown as narrow zones, in which the probability declined very rapidly from the most likely location. *Fuzzy borders* appeared as zones of more or less predictable probability pattern having cross-sectional profiles similar to bell-shape familiar from the Normal distribution density function. *Alternative borders* were the most unpredictable form of basin limits, in which the border may have two or more alternative locations. It is worth to notice, that alternative borders existed even if the DEM error was considered to be very small. Thus these areas can be considered as critical regions, which may need additional field survey for reliable watershed delineation.

The results of error propagation in slope and aspect calculations were in agreement with the results of watershed delineation. The standard deviation of the vertical DEM error appeared to have a major role in error propagation of both local derivatives within the tested spatial autocorrelation ranges. The higher variance in DEM elevations resulted in higher variance in slope/aspect calculations. Opposite to watershed delineation, the increase of spatial autocorrelation range increased the slope/aspect variation only when the range was equal or less than 20 meters with the predefined spatial autocorrelation ranges. When the spatial autocorrelation range increased beyond this limit, it actually caused the decrease in slope/aspect standard deviation. This is the result caused by the size of the slope/aspect calculation window. Also the average slope has a major role in local derivatives standard deviation. Even with the smallest DEM error parameters, the calculation of aspect was very uncertain in low-lying areas of the island. Also slope has the highest variation in flat terrain, but the effect is remarkably smaller than in aspect calculation.



**Figure 2.** DEM error propagation in watershed delineation. Shades represent the confidence limits of the probable watersheds. The maps show a 2 × 2 subset of all 16 scenarios of probabilistic drainage basin delineations generated in the study.

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