

WHICH IS THE BEST PROJECTION FOR THE WORLD MAP?

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Abstract

The main goal of the paper is devising of an objective procedure serving for assessment of cartographic projections. The newly proposed indicator of projection's quality "distortion characterization Q " was defined as the percentage ratio of the area represented in the map with permissible distortion to the area of the whole world. Maximum angular distortion 40° and 1.5 multiple of the smallest areal distortion were used as the distortion limits. Then a hundred conventional projections were arranged into the sequence list in compliance with Q . The best projections have Q over 80 %.

1. INTRODUCTION

Cartographic projections are the methods that unambiguously assign to every point on the Earth's surface just one point in the plane of the map. Application of different projections gives for small regions insignificant deviations, sometimes smaller than inaccuracy of map drawing. The larger is the represented area, the more the reality is distorted in the map. The largest distortion arrives in one-sheet world maps.

A whole number of projections has been used in geographical atlases: conventional projections prevail at the present. Cylindrical projections are quite rare now because of the poles equally long like the equator or standard parallels. As a rule publishers of atlases give no reasons for their choice of a certain cartographic projection and users little dream to occupy their minds with the question whether is the projection of a good or poor quality. Choice of the best projection cannot have a definite solution. It always depends on demands which the projection has to satisfy: properties (e.g. equal-area), shape of the graticule (e.g. straight line parallels) and shape of the Earth in the map (e.g. ellipse). However, it would be necessary to have at disposal some sequence list, reflecting projections' quality on the basis of an accurate criterion. Then we could search the best projection fulfilling beforehand given requirements in the sequence list.

The aim of this work is a proposition of such accurate criterion and then compilation of a sequence list of one-sheet world map projections according to criterion mentioned above. Conventional uninterrupted projections are taken into consideration only.

2. DISTORTION

Accurate criteria for evaluation of cartographic projections are based on numerical values of local distortions in the infinitely small neighbourhood of individual points in a map.

Distortion refers to lengths, areas and angles, and its value is changing point by point continuously.

The length distortion k is the ratio between the infinitely small length in the map and the corresponding length on the reference globe. The areal distortion K is the ratio between the definitely small area in the map and the corresponding area on the globe. The angular distortion 2ω is the difference between the angle measured in the map and the corresponding

angle on the globe. Besides that the values of length and angular distortion depend on the direction, too. Relating to lengths, length distortion along meridians k_p and length distortion along parallels k_r are taken into consideration only. Relating to angles, maximum angular distortion 2ω in the worst direction is accepted.

Connecting line of the neighbouring points with the same distortion is called the distortion line. The distortion lines in conventional projections are usually formed by complicated curves, symmetric in compliance with straight central meridian and the equator. Areal distortion lines are straight in pseudocylindrical projections only.

Computation of distortions for conventional projections is very sophisticated. Knowledge of projection's formulae is necessary for each projection. Distortion's formulae are derived on the basis of projection's formulae through partial derivatives. Then geographical co-ordinates of an particular point are substituted into distortion's formulae and the distortion in this individual point is computed.

The procedure includes derivation of formulae for computation of length distortion along meridians k_p , length distortion along parallels k_r , and deviation τ , which is equal to the difference of the angles between meridians and parallels in the map from 90° :

$$k_p = \sqrt{\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2} \quad k_r = \sqrt{\left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2} : \cos \varphi$$

$$\operatorname{tg} \tau = -\left(\frac{\partial x}{\partial \varphi} \cdot \frac{\partial x}{\partial \lambda} + \frac{\partial y}{\partial \varphi} \cdot \frac{\partial y}{\partial \lambda}\right) : \left(\frac{\partial x}{\partial \lambda} \cdot \frac{\partial y}{\partial \varphi} - \frac{\partial x}{\partial \varphi} \cdot \frac{\partial y}{\partial \lambda}\right)$$

General equations are valid for computation of areal distortion K and maximum areal distortion 2ω :

$$K = k_p \cdot k_r \cdot \cos \tau \quad 2\omega = 2 \operatorname{arctg} \cdot \frac{1}{2} \sqrt{\frac{k_p^2 + k_r^2}{K} - 2}$$

3. EVALUATION CRITERIA

Quality of an cartographic projection as a whole can be evaluated by one number in conformity with following global criteria:

- a) Comparison of finitely large distances. Projection's quality is evaluated by the mean percentage deviation between distances measured in the map and orthodromic distances on the reference globe.
- b) Total criteria. Mean values of areal, length and angular distortions are established. These heterogeneous values are replaced by means of multicriterial evaluation by one number named "global mean distortion" (CAPEK 1997).
- c) Extreme criteria. Projections are evaluated in compliance with the extent of area where areal and angular distortions do not exceed prescribed maximum limits (CAPEK 1998).

These particular extreme criteria were used for evaluation of quality of conventional projections in this work. The distortion characterization Q was established as an indicator of projection's quality. Q is the percentage ratio of area, where neither areal nor maximum angular distortion does not exceed permissible limits, to the whole area of the reference globe. These limits must be unified for all projections otherwise they could not be comparable one another. The permissible maximum angular distortion is marked by the symbol $2\omega_{\max}$. The

permissible areal distortion K_{max} is n -multiple of the smallest numeric value K_{min} in the map (if areal distortion decreases from the equator to the poles, the reverse is true).

Experimental works with different limits of areal and maximum angular distortions had been performed before definitive permissible distortion limits $2\omega_{max}$ and K_{max} were set down.

Values $2\omega_{max}$ (equal gradually to 30°- 35°- 40°- 45°- 50°) and values n (equal gradually to 1.3 - 1.4 - 1.5 - 1.6 - 1.7) were substituted. These limits had been combined for different projections one another. Then characterizations Q were computed and sequences were compiled (CAPEK - FORSTOVA 1999).

There were no considerable differences between the sequences compiled of combinations 35°- 40°- 45° with 1.4 - 1.5 - 1.6. The average sequence of combination $2\omega_{max} = 40^\circ$ and $n = 1.5$ (areal distortion does not exceed 50 % of minimum value). Therefore the value $2\omega_{max} = 40^\circ$ as the permissible maximum angular distortion limit and the value $K_{max} = 1.5 K_{min}$ as the permissible areal distortion limit were accepted.

4. DISTORTION CHARACTERIZATION Q

The distortion characterization Q was calculated for about hundred projections that are applicable for one-sheet uninterrupted world map. Source materials for this work were considerably heterogeneous:

- a) distortion formulae of individual projections
- b) projections` formulae
- c) tables of distortion values for limited amount of points
- d) distortion lines drawn into the graticule of given projection

Distortion formulae were either known or newly derived from projections` formulae for approximately a third of all projections. Computer processing was used for computation of the distortion characterization Q for these projections. Values of distortion were computed in graticule intersection points with 1° interval. Each point was weighted by respective graticule trapezoid $1^\circ \times 1^\circ$ area on the sphere. The total area with the permissible distortion was gained as a sum of all graticule trapezoids where the values of areal and angular distortion did not exceed the permissible limits. Computer processing was made by my colleague Jana Forstova to whom belongs my highest appreciation!

Precision of results was influenced by graticule`s interval: for example Putnins P3` projection has value of $Q = 78.6 \%$ when graticule trapezoid $1^\circ \times 1^\circ$ is used, whereas graticule trapezoids $5^\circ \times 5^\circ$ cause rising of Q by 0.8 %. The difference between Q calculated on sphere and on ellipsoid makes 0.1 % only.

Cartometric processing taking advantage of distortion lines was used for the rest of projections. Border distortion lines with permissible distortion limits $2\omega_{max}$ and K_{max} were drawn into every projection`s graticule; tables of distortion values or former published distortion lines with round values were adopted as a base. Tabular values of graticule trapezoid areas $10^\circ \times 10^\circ$ or rather $5^\circ \times 5^\circ$ were used for calculation.

Imprecision caused by approximation of incomplete trapezoids reached about one tenth of trapezoid area. It means that values of Q were changed approximately by 1 % , but in the reality positive and negative differences equalized one another. Graticule trapezoids are smaller near the poles. Therefore the precision of Q is higher for the best projections where the distortion lines occur nearer to the poles and lower for the worse projections with distortion lines far from the poles.

Both computer and cartometric processing were made for 30 projections because of comparison of Q values of the same projection. Differences between both of methods were smaller than 0.5 % in 14 cases and higher than 1 % in 7 cases. Only one projection had the

difference over 2 % but poor approximation of graticule trapezoids during cartometric processing was not responsible for it. The fault lies in dubiously plotted distortion lines in cases where calculations of Q are based only on published pictures of distortion lines without possibility of their verification.

The characterization Q was also assessed for several interrupted projections with the aim comparing them with other projections. Even though values of Q are higher and distortions smaller, interrupted projections resulted in unnatural picture of splitted world.

The final sequence list of evaluation of conventional projections according to characterization Q is contained in the table 1.

Table 1. Sequence list of conventional projections according to characterization Q (in %).

place	projection	Q set by	
		computer	cartometry
1	CNIIGAiK (or TsNIIGAiK) 1950 (= Ginzburg V)		84.7
2	CHISI (or KhISI)		83.4
3	Taic (or Taich)		83.2
4	Robinson		82.6
5	Hufnagel 10		82.1
6	Kavrajiskij (or Kavrayskiy) VII	82	82
7	Eckert IV	81.9	82.5
7	Ortelius = Apian I		81.9
9	Winkel III $\varphi_0 = \pm 40^\circ$	81.3	81.7
10	Urmajev (or Urmayev) II	81	79.5
11	Winkel II	80.9	81.1
12	Winkel III $\varphi_0 = \pm 50.46^\circ$	80.7	80.6
13	Wagner V	80.6	80.9
14	Putnins P1`	80.4	79.5
14	Wagner VI	80.4	79.5
16	Hufnagel 9		80.1
17	Wagner VIII		80
18	Eckert III	79.9	79.8
19	Hufnagel 7		79
19	Putnins P3`	79	78.6
21	Hufnagel 11		78.5
22	Hill		78.2
23	Aitow - Wagner		77.9
23	Wagner III	77.9	77.7
25	Michajlov II		77.8
26	Hufnagel 12		77.2
27	CNIIGAiK 1954 (= Ginzburg VII)		77
28	Eckert V	76.7	76.1
29	Apian II		76.5
29	Hufnagel 4		76.5
31	Wagner II	76.4	75.8
32	Putnins P2` = Wagner IV	76.3	75.3
33	Tobler 10		75.9
33	Urmajev I		75.9
35	Winkel I	75.7	76.1

36	Peters VII		75.4
37	Werenskiold III		75.3
38	Hufnagel 3		75.2
39	Hufnagel 2		74.9
40	Putnins P5`	74.7	
41	Hojovec		74.2
42	Kavrajskij V	74.1	74.2
43	Wagner VII		74
44	Werenskiold I		73.6
45	CNIIGAiK BSE (= Ginzburg VI)		73.2
45	Grygorenko (= STWP)		73.2
47	Putnins P4`	72.4	73.6
47	Sin and tg series 1		72.4
49	CNIIGAiK 1939-1949 (= Ginzburg IV)		72.3
50	Kavrajskij VI = Wagner I	71.2	70.6
51	Erdi Krausz	70.9	70.6
52	Mollweide	70	70.4
53	Nell - Hammer II		69.5
54	CNIIGAiK 1939		69.1
55	Eckert VI	68.9	68.7
56	Putnins P1	68.8	66.4
57	Putnins P6`	68	
58	Mc Bryde - Thomas IV		67.7
59	Sin and tg series 2		67.3
60	Aitow		66.6
61	Mc Bryde - Thomas V		65.9
62	Sin and tg series 3		65.3
63	Eckert I	64.7	65.8
63	Putnins P2	64.7	63.1
65	Hammer		64.3
66	Grinten III		64
67	CNIIGAiK 1944 (= Ginzburg VIII)		63.4
67	Mc Bryde - Thomas III		63.4
69	Putnins P3	62.9	61.5
70	Adams quartic		62
71	Baranyi IV		61.5
72	Eckert VII		61.4
73	Hammer - Solovjev		61.3
74	Briesemeister		61.1
75	CNIIGAiK with oval distortion lines (= Ginzburg III)		60.5
76	Craster = Putnins P4	60.3	61.5
77	Boggs		59.8
78	Michajlov I	58.5	
79	Sanson	57.1	57.6
80	Grinten I		56.5
81	Eckert II	56.2	57.3
82	Bourdin		54.5
83	Putnins P6	54.4	

84	Putnins P5	53.6	
85	Bonne		53.3
86	Sin and tg series 7		52
87	Baranyi II	51.5	
88	Fournier I	50.1	
89	CNIIGAiK 1944 (formulae by Canters - Declair)	49.5	
89	Werner - Stab		49.5
91	American polyconic (= Hassler)		47.9
92	Foucaut		44.3
93	Sanson modified		43.4
94	Prepetit - Foucaut		42
95	Guyou	37.2	37.1
95	Peirce	37.2	37.1
97	Rectangular polyconic (= War Office)		35.5
98	Collignon I		32.4
99	Lagrange		27
100	August		20.7
Interrupted projections:			
	BSAM		90
	Goode homolographic I		89.3
	Goode - Sanson		85.5

5. ANALYSIS OF THE BEST PROJECTIONS

Values of characterization Q are higher than 75 % for 39 projections and even higher than 80 % for 17 of them. Let us fasten our attention just of these 17 projections. They are predominantly pseudocylindrical, some of them are pseudoazimuthal and polyconic. Four projections only are equal-area. The equator and central meridian are straight lines, the poles are straight or curved lines.

Places No 1 - 5 are occupied by projections where neither distortion formulae nor projection's formulae were available (mostly even they do not exist). Characterizations Q were gained only on the basis of distortion lines printed into graticules or constructed according to tables. Polyconic projection CNIIGAiK 1950 (=Ginzburg V) was empirically derived by its Russian author by means of so called "sketches of the map graticule" (in Russian "metod po eskizu setki"). This projection is reliable, many years used and worth to be recommended. Far from it are polyconic projections CHISI and Taic, designed by Ukrainian cartographers before the World War II. They have been never used and are not recommendable. Big differences between local distortion values established experimentally by graphic way and published distortion lines were detected. That makes very dubious also Q values computed in compliance with these distortion lines. On the contrary, pseudocylindrical projection designed by American cartographer Robinson with the goal keeping of shapes as far as possible, is in common usage and we can recommend it. German equal-area pseudocylindrical projection Hufnagel 10, created in the year 1989, has not been used up to now. However, it is worthy of thorough examination in future.

Projections on places No 6 - 15 (with exception of pseudocylindrical Ortelius projection) were evaluated by computer processing and their characterizations Q are objective. Two of these projections are pseudocylindrical with elliptic meridians and straight parallels: both have Q almost the same value of 82 %. Kavrajiskij VII projection is arbitrary with equidistant central

meridian and standard parallels $\varphi_0 = \pm 35.5^\circ$, the equator is 0.866-times reduced. Areal distortion $K = 0.866$ and maximum angular distortion $2\omega = 8.2^\circ$ in the centre of map. Parallels and the central meridian are divided proportionally. Eckert IV projection is equal-area with standard parallels $\varphi_0 = \pm 40.5^\circ$. Parallels are divided proportionally, parallel spacing decreases from the equator to the poles. The equator is 0.844-times reduced. Maximum angular distortion in the centre of map $2\omega = 19.3^\circ$. While Kavrajiskij VII occurs very rarely (only in a few Russian atlases), Eckert IV is used much more. Both projections can be fully recommended.

Pseudoazimuthal arbitrary projection Winkel III with standard parallels $\varphi_0 = \pm 40^\circ$ or $\pm 50.46^\circ$ is justly widespread in the whole world. Meridians and parallels are curved lines. Central meridian is equidistant in both cases. Areal distortion and shortening of the equator simultaneously equal 0.883 when $\varphi_0 = \pm 40^\circ$ or 0.818 when $\varphi_0 = \pm 50.46^\circ$, maximum angular distortion in the centre of the map $2\omega = 7.1^\circ$ when $\varphi_0 = \pm 40^\circ$ or 11.5° when $\varphi_0 = \pm 50.46^\circ$. The shape of Winkel III is very similar to the projection CNIIGAiK 1950.

It is very surprising that the next places of sequence list are occupied by almost unused projections, which are known only to very limited number of cartographers. It relates to following pseudocylindrical projections: Urmajev II, Winkel II, Wagner V, Putnins P1` and Wagner VI. We can recommend Winkel II, Wagner V and Putnins P1` without any doubts. The said projections - together with formerly named Robinson, Kavrajiskij VII, Eckert IV and Winkel III - have also very favourable values of global mean distortion (CAPEK 1997). The just mentioned indicator is worse for Wagner VI and rather poor for Ortelius; it was not computed for Urmajev II, Hufnagel 10 and polyconic projections CNIIGAiK 1950, CHISI, Taic. Pseudocylindrical equal-area Hufnagel 9 and pseudoazimuthal non equal-area Wagner VIII have Q near 80 %, global mean distortion is not known.

6. CONCLUSION

What is the final answer for the question in the title of this article? With regard to precision and reliability of the methods used for computation of distortion characterizations Q and with taking global mean distortion into consideration, we can state that the best projections for one-sheet uninterrupted map of the world are:

- a) CNIIGAiK 1950 and Robinson from among projections where Q was established by cartometric processing
- b) Kavrajiskij VII, Eckert IV, Winkel III, Winkel II, Wagner V and Putnins P1` from among projections where Q was established by computer processing

The other projections with characterization Q about 80 % have not been verified enough for safe recommendation.

As far as the remaining projections contented in the sequence list (Q moves from 20 % to 90 %), it depends on cartographer's requests: conformality, point poles or elliptic shape of the world, perhaps. Each cartographer can choose the projection satisfying his demands and simultaneously having the highest possible value of characterization Q .

Table 2. Projection formulae (radius of a globe $r = 1$)

Eckert IV	$2\psi + 4 \sin \psi + \sin 2\psi = 7.141593 \cdot \sin \varphi$ $x = 0.422238 \cdot \lambda(1 + \cos \psi)$ $y = 1.326500 \cdot \sin \psi$
Kavrajiskij VII	$x = 0.866025 \cdot \lambda \cdot \sqrt{1 - 0.303964 \varphi^2}$ $y = \varphi$

Putnins P1`	$x = 0.947449 \cdot \lambda \cdot \sqrt{1 - 0.303964 \varphi^2}$	$y = 0.947449 \cdot \varphi$								
Wagner V	$2\psi + \sin 2\psi = 3.008957 \cdot \sin(0.885502\varphi)$ $x = 0.909771 \cdot \lambda \cdot \cos \psi$	$y = 1.650142 \cdot \sin \psi$								
Winkel II	$x = \frac{\lambda}{2} \cdot \left(0.636620 + \sqrt{1 - 0.405285 \varphi^2} \right)$	$y = \varphi$								
Winkel III	$\cos \Delta = \cos \varphi \cdot \cos \frac{\lambda}{2}$ $x = \frac{1}{2} \cdot (2\Delta \cdot \sin \Lambda + \lambda \cdot \cos \varphi_0)$	$\cos \Lambda = \frac{\sin \varphi}{\sin \Delta}$ $y = \frac{1}{2} \cdot (\Delta \cdot \cos \Lambda + \varphi)$								
CNIIGAiK 1950	co-ordinates were published in GINZBURG – SALMANOVA 1957									
Robinson	(x means the length of the parallel φ)									
φ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
x	266.63	265.40	261.88	255.96	245.72	231.41	212.93	191.60	165.66	141.89
y	0.00	16.77	33.54	50.31	67.05	83.52	99.34	117.07	127.04	135.27

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