ELLIPSOIDAL GNOMONIC PROJECTION BY MEANS OF DOUBLE PROJECTION

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［Abstract］ According to the country standard of the peoples republic of china-*the cartographic specification for chinese nautical charts* practised from 1999-05-01.It is requested that Gnomonic Projection be used if over 60% of the charted area is $\phi>75°$ .In this paper,the gnomonic formula from the earth ellipsoid to the surface of projection is derived by double projection method,the three important characterictics and application of the Ellipsoidal Gnomonic Projection are described.

［Key words］ Earth ellipsiod Great ellipse Hyperbola of common focus Ellipsoidal Gnomonic Projection

Ⅰ.Ellipsoidal Gnomonic Projective formula

Elliposidal Gnomonic Projective formula can be established by means of double projection.

Step 1:describe the earth ellipsoid on the surface of geocentric latitude. The formula reads:

$$\left.\begin{aligned} \mathrm{tg}\,u&=(1-e^2)\mathrm{tg}\,\phi \\ \omega&=\lambda \end{aligned}\right\} \qquad (1)$$

Where (u, ω) is the geodetic coordinate corresponding to the globe coordinate (λ , ϕ)， e is the first eccentricity of ellipsoidal.

If we make the radius of globe $R=r_0$(r_o is ellipsoidal motional radius at point r_o of tangency), then the great ellips of ellipsoid is projected into great circle.

Step 2:Project the geocentric latitude sphere to plane of map by means of centre perspective projection.We assumed that the geocentric latitude (ϕ_0) of point of tangency is u_0 and this point of tangency is the origin of plane rectangular coordinate. Then the projective formulas reads [2] :

$$\left.\begin{aligned} x&=r_0(\mathrm{tg}u-\mathrm{tg}u_0\cos\omega)/(\mathrm{tg}u\,\mathrm{tg}u_0+\cos\omega) \\ y&=r_0\sec u_0\sin\omega/(\mathrm{tg}u\,\mathrm{tg}u_0+\cos\omega) \end{aligned}\right\} \qquad (2)$$

If we substitute(1) into (2),we get

$$\left.\begin{aligned} x&=r_0(1-e^2)(\mathrm{tg}\,\phi-\mathrm{tg}\,\phi_0\cos\lambda)/\left[(1-e^2)^2\,\mathrm{tg}\,\phi\,\mathrm{tg}\,\phi_0+\cos\lambda\right] \\ y&=r_0\left[1+(1-e^2)^2\mathrm{tg}^2\,\phi_0\right]^{1/2}\sin\lambda/\left[(1-e^2)^2\mathrm{tg}\,\phi\,\mathrm{tg}\,\phi_0+\cos\lambda\right] \end{aligned}\right\} \qquad (3)$$

Formula(3) is just the Epllisoidal Gnomonic projective formula according to the method of double projection.

Because of using double projecton, the accuracy is strictly controlled ,so that projection deformation is very little in the first operation. This is also the important contribution of liberature(3).Let spherical surface minify to the point of tangency which passing Gnominic projection, that is to say that the spherical surface is superimposed with the ellipsoid surface at the point of tangency,then the radius of globe become a motional radius which connecting the point of tangency of ellipsoid with the ellipsoidal center. For large and medium scale map which use Gnomonic Projection, distortion of distance and azimuthal angle can omitted account when describe the ellipsoid surface to the geocentric latitude spherical surface.The distanse distortion formula of the ellipsoid surface Gnomon Projection by method of doubling projection is the same as of the general perspective azimuthal project of center of globe[4] . The distance ratio of latitude and longitude reads:

$$\left.\begin{aligned} m&=(1-\cos^2 u_0 \sin^2 \omega)^{1/2}/k^2 \\ n&=(k^2+\cos^2 u_0 \sin^2 \omega)^{1/2}/k^2 \end{aligned}\right\} \tag{4}$$

Where $k=\sin u \sin u_0+\cos u \cos u_0 \cos \omega$

The largest angle distortion(W) of Ellipsoid Gnomon Projection can be defined as:

$$\mathrm{Sin}(\omega/2)=(1-k)/(1+k) \tag{5}$$

Area scale(p) expresses is:

$$p=1/k^3 \tag{6}$$

Ⅱ.Three important character of Ellipsoid Gnomon Project

Frist character:Any great ellipse on ellipsoid surface is projected to Straight line.Proving as follows: devide the first equation of formula(3) by Second

$$x/y=(1-e^2)(\mathrm{tg}\,\psi-\mathrm{tg}\,\psi_0\cos\lambda)/[1+(1-e^2)\mathrm{tg}^2\,\psi_0]^{1/2}\sin\lambda \tag{7}$$

Then we find that

$$\mathrm{tg}\,\psi=(x/y)[1+(1-e^2)^2\mathrm{tg}^2\,\psi_0]^{1/2}\sin\lambda/(1-e^2)+\mathrm{tg}\,\psi_0\cos\lambda \tag{8}$$

Substitude(8) into the second formula(3) ,we get the latitude projection equation.

$$y=\mathrm{tg}\,\lambda[r_0-x(1-e^2)\mathrm{tg}\,\psi_0]/[1+(1-e^2)^2\mathrm{tg}^2\,\psi_0]^{1/2} \tag{9}$$

Formula(9) illustrates that latitude on ellipsoidal surface becomes straight line by projection.

We can get anticaculative formula from formula(9)

$$\mathrm{tg}\,\lambda=[1+(1-e^2)^2\mathrm{tg}^2\,\psi_0]^{1/2}y/[r_0-x(1-e^2)\mathrm{tg}\,\psi_0] \tag{10}$$

The great ellipsoidal equation of ellipsoid surface is known:

$$(1-e^2)\mathrm{tg}\,\psi=\mathrm{ctg}A_0\sin(\lambda-\lambda_0) \tag{11}$$

Where A_o is the geodetic azimuth of the intersection point where the great ellipsoid intersect with the equator. λ_0 is the geographic longitude of this point. Dispreading sinusoidal function from formula(11).

$$(1-e^2)\mathrm{tg}\,\psi=\mathrm{ctg}A_0(\sin\lambda\cos\lambda_0-\cos\lambda\sin\lambda_0) \tag{12}$$

Substitude (8) into (12).

$$(1-e^2)\mathrm{tg}A_0\ \{(x/y)[1+(1-e^2)^2\mathrm{tg}^2\,\psi_0]^{1/2}\mathrm{tg}\,\lambda/(1-e^2)+\mathrm{tg}\,\psi_0\}$$

$$= tg\,\lambda \cos\lambda_0 - \sin\lambda_0 \qquad (13)$$

Substitude (10) in (13), after simpliflating ,we get

$$[1+(1-e^2)^2 tg^2\,\psi_0]^{1/2} y\cos\lambda_0$$

$$= r_0[(1-e^2)\,tgA_0 tg\,\psi_0+\sin\lambda_0]-x\,[(1-e^2)\,tg\,\psi_0\sin\lambda_0+ tgA_0] \qquad (14)$$

Then such solution is described by:

$$y=\{\,x[tgA_0-(1-e^2)\,tg\,\psi_0\sin\lambda_0]+r_0\,[(1-e^2)\,tgA_0 tg\,\psi_0+\sin\lambda_0]\,\}$$
$$/\{\,[1+(1-e^2)^2 tg^2\,\psi_0]^{1/2}\cos\lambda_0\,\} \qquad (15)$$

As it is known from formula(15). The great ellipsoid of ellipsoid gnomon projection is represented as a straight line. The slope of which is

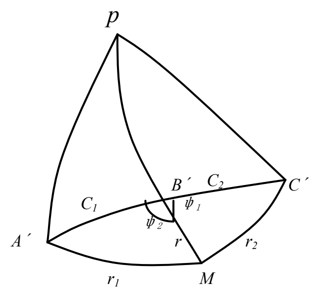
$$k=[tgA_0-(1-e^2)\,tg\,\psi_0\sin\lambda_0]/\{\,[1+(1-e^2)^2 tg^2\,\psi_0]^{1/2}\cos\lambda_0\,\}$$

Second character: It becomes two group plane hyperbola family when two group ellipsoid hyperbola family with point of tangency as polar are projected. Proving as follow:

We suppose that three station locate on the earth ellipsoid as A、B and C.The points they project to the geocentric latitude globe are A´ (u_1,w_1) B´ (u_2,w_2) and C´ (u_3,w_3). For the great ellipsoid is projected to be the great circle on the geocentric atitude surface, the maximum change value of azimuthal angle is 0.´6[5].It can be omitted for mapping. So the spherical azimuth (α) is regarded as the geodetic azimuth (A). So that the spherical surface hyperbola of common focus with point (B´) of tangency as the acting pole can be regarded as the projection of the ellipsoidal hyperbola of common focus with point (B) of tangency as the acting pole.

As show in chart, we suppose that the spherical surface base curve BA=C_1 ,BC=C_2, The distance from any point M(u,w) to spherical surface are separetely r_1,r,和 r_2. The spherical distance difference are separately $a_1=r_1-r$,$a_2=r_2-r$. Then in spherical triangle A´M B´和 B´MC´, According to cosine theorem we can get

$$\left.\begin{aligned}\cos a_1&=\cos c_1+tg\,r(\sin a_1+\sin c_1\cos\psi_1)\\ \cos a_2&=\cos c_2+tg\,r(\sin a_2+\sin c_2\cos\psi_2)\end{aligned}\right\} \qquad (16)$$



Equation(16) is the formula of two spherical surface hyperbola of common focus with point (B´) as the acting pole. Their focal distance are separately C_1 andC_2.

Transformating expression (16) into expression(17)

$$\left.[\cos a_1-\cos c_1]/\sin a_1=tg\,r(1+\sin c_1\cos\psi_1/\sin a_1)\right\} \qquad (17)$$

$[\cos a_2-\cos c_2]/\sin a_2=\mathrm{tg}r(1+\sin c_2\cos\psi_2/\sin a_2)$

Supposing

$$\left.\begin{aligned}p_1&=[\cos a_1-\cos c_1]/\sin a_1, l_1=\sin c_1/\sin a_1\\ p_2&=[\cos a_2-\cos c_2]/\sin a_2, l_2=\sin c_2/\sin a_2\end{aligned}\right\}\quad(18)$$

Then we get the spherical surface hyperbola of common focus with polar coordinate form express [6] .

$$\left.\begin{aligned}p_1&=\mathrm{tg}r(1+l_1\cos\psi_1)\\ p_2&=\mathrm{tg}r(1+l_2\cos\psi_2)\end{aligned}\right\}\quad(19)$$

Let the spherical surface hyperbola of common focus of formula(19) project on the gonmon projection plan with point(B ´) as the pint of tangency.Such polar coordinate of spberical hyperbola point M(u,w) is expressed as follows.

$$\left.\begin{aligned}p'&=\mathrm{tg}r\\ \alpha'&=\alpha_{B'A'}-\psi_1\end{aligned}\right\}\text{ or }\left.\begin{aligned}p''&=\mathrm{tg}r\\ \alpha''&=\alpha_{B'C'}-\psi_2\end{aligned}\right\}\quad(20)$$

Where $\alpha_{B'A'}$ and $\alpha_{B'C'}$ separatly represent the spherical azimuth of B´A´and B´C´.Representation of formule(19) is hyperbolic lattice of common focus that point(B) of tangency is regarded as pole and starght line B´A´ and B´C´ are axes.

$$\left.\begin{aligned}p'&=p_1/(1+l_1\cos\psi_1)\\ p''&=p_2/(1+l_2\cos\psi_2)\end{aligned}\right\}\quad(21)$$

It is known from literature [7] that in gnomon projection with station B is regarded as point of tangency when latitudind zone is within 30° (or scale is greater than 1∶3 330 000), distanse deformation from ellipsoid surface to geocentric latitude spherical surface may be pretermitted.In the same time, deformation of Distance and azimuth in zone are pretermitted.So two group hyperbola of common focus on spherical surface is regarded as hyperbola of common focus on ellipsoidal surface. According to formula (21), It is proved that two group hyperbolic lattice of common focus on ellipsoidal surface of point (B) is regarded polar is projected into two group plane hyperbolic lattice of common focus.

Third character:The azimuth from point of tangedcy to any point is equal to the geodetic azimuth.

This character can be proved by the double projection of point of fangency, from the ellipsoid surface to the geocenfric latitude surface, the azimuth of the spherical surface azimuth of the point of tangency can be regarded as the geodetic azimuth, or $\alpha=A$.When the earfh surface is projected to the tangency surface, according to characters of aximuth project, the angles formed by the longtitudes passing the point of tangency and every vertical chart is changeless or $\delta=\alpha$, So the azimuth from the point of tangency to any point on the map is equal to the geodetic azimuth or $\delta=A$

Ⅲ.Application of ellipsoid surface gonmon project in navigational chart.

The importance of ellipsoid surface gnomon project in chart mapping is only after Merctor Project. It is Well known that the great ellipse line(it is regared as geodesic line) is the shortest distance between two point on ellipsoid surface. When navigating along Merctor track, the ship need to navigate more. It is the best way for the ship to navigate along the great ellipse for saving time and cost to arrive destiation. Because the great circle in earth gnomon projection is regarded as straight line [8], the great allipse in ellipsoid surface gnomon is regarded as straight line. On this chart great ellipse navigation line of between two point is ploted with straight line. In this way ship may navigate along great ellipse.

Second character of Ellipsoid gnomon is that two group ellipsoid hyperbola family of point of tangency is regard as polar is projected into two group plane hyperbola family.It is used for making radio hyperbola navigational chart or drawing hyperbola location grid on gnomon project chart for ship locating when navigate in the ocean.

According to third character of ellipsoid gnomon project or azimuth from point of tangedcy to any point is same as geodetic azimuth.it is used in making special chart that radar station, view station and missle base place use where is situated on point of tangency. It ia used mainly for measuring angle and locating, too [9].

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