Research on System Space Mathematical Basis of Digital Earth

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[Abstract]

Being affected by the basis theory and method of traditional map projection, nearly all GISs take certain map projection as their own spatial mathematical basis, which is feasible in a partial area and specific use. Along with the appearance of “ Digital Earth”, the GIS cover has been extending from a partial area to a large region or even the whole earth, howe’er the mathematical basis of GIS has not been changed and developed correspondingly, which can not meets Digital Earth requirements of the multi-resolution and continual visualization. The paper extends the concept of map projection to the projection between two curve surfaces instead of the traditional projection from a curved surface to a plane. It also puts forward a practical model for the map projection model of Digital Earth whose plane expression adopts the cylindrical equidistant projection and measure space is ellipsoidal coordinate system, which accords with the separation characteristic of GIS’s expression from measure space in the computer environment.

Keywords: Digital Earth; map projection; mathematical basis; GIS

1. Preface

The problem of spatial mathematical basis has been encountered by both large-scale GIS and spatial digital products, theoretically and practically. It is also a basic problem in the development of the whole geo-information science. Nowadays, GIS academic circle has cared greatly about the problem.

At present, a variety of GISs were set up like bamboo shoots after a spring rain. There are collections and applications of the terrestial digital products progressively. Therefore, it is an urgent task to solve the problem.

2. The limitation of the spatial mathematical basis for GIS nowadays

Map projection is defined as a topological transformation between two 2D fields in order to present features on spheroidal earth in a limited plane map, different map projections cause different errors. In general speaking, which map projection should be chosen, we always consider three elements of map projection, i.e., map use, scale and the position and shape of the area. Map projection is the space mathematical basis of maps.

GIS come of Cartography, maps is the major data source of GIS. At present, most GISs take over the concept of map projection, and select different referential system and projection suitable for the area and scale as own space mathematical basis. Under these circumstances, ordinary GISs mainly use base data with a single general scale, before other resolution data was bring into the system, there was a necessary transform between different scales. The small scale GISs use the map projection suitable for the small scale as own space mathematical basis, and the large scale GISs use the large scale map projection. There are some limitations for the former, the mathematic basis distorts seriously, the data integration precision after transformation is not good, and the precise of the measure space is fallibility. As a result, the special analysis is difficult to be done. For the later, the large scale GISs is only used in a partial area. If used for the large-scale area or the whole earth, it would bring the phenomenon of the graphic discontinuity. Therefore, the global spatial analysis is unable to be done.

The object of the large-scale GIS is a huge area with certain broadness or even the whole earth. The general three elements of map projection can't be defined properly. So, from the view of the basic demands for Digital Earth, the continuing multi-resolution of the whole earth and their various applications, the traditional selection of map projection for the large-scale GISs is no more suitable.

In China, most GISs adopt the map projection system with related resolution in accordance with the state basic succeeding scales topomaps as own space mathematical basis. That is, the Gauss projection is applied by the large scale GISs, and lambert projection with the small scale GISs. From the following analysis we can realize deficiencies for large-scale GIS caused by Gauss projection.

For Digital Earth and large-scale GISs, the basic deficiency in applying Gauss projection is that it cannot realize entire continuing visualization, both for sliced projection or broad zone projection, especially for the latter the deformation is too big and complex, and that will make the following work more difficult and cause bad affect.

Essentially, Gauss projection is not suitable for multi-resolution orientation construction and expression. In Gauss projection system, it is rather complex to define the accurate length and its measurement, and it is also difficult in application. The GIS of an area faces many dynamic variations, such as the expansion and coincidence of boundary, and interchange with outside areas, etc. Therefore, large-scale GISs are not necessarily to continue to use Gauss projection as their spatial mathematical basis.

Furthermore, the fact that different GISs use self-defined different spatial referential systems is no good for the development of GIS and the object of Digital Earth. Therefore, it is very clear that a new spatial mathematical basis for this purpose is urgent and necessary. Thus the spatial data and information will have a highly common data base of sharing.

3. Spatial mathematical basis suitable for large-scale GIS

The most suitable referential system for large-scale GIS or Digital Earth is the geodetic coordinate system, which can be easily and solely used to depict the position of any point in the spatial zone. At present the most widely used coordinate systems are the geocentric coordinate system and the geodetic system. These two systems are widely used in geodetic science. However, there exists a small difference. For large-scale GIS and Digital Earth, this difference can be neglected, and the correction can be calculated precisely, if necessary.

The aim of large-scale GIS and Digital Earth causes the study field of this topic fixed on the whole China territory or even more larger area. In addition to the above mentioned ellipsoidal coordinate system, in order to realize mutual information application and operation, the standardization of map projection is very important.

3.2 The map projection most suitable for large-scale GIS and Digital Earth

3.2.1 Basic demands for the standard of the map projection suitable for Digital

Earth

1) Simple structure, rigorous basis, precise and convenience in measurement. Map projection is the spatial mathematical basis of maps. Map, as the 2-D symbolic model of geographic information on its main projection direction, defined a 2-D plane in certain area and certain precise measurement. The users of maps can see, measure, realize, analyse and apply the geographical information in more wider area and more precise.

2) Large area with multi-resolution and continual visualization for even the whole world. As Professor Goodchild rightly says [1], In DE, the range of scales implied is over at least four orders of magnitude, from a resolution of 10km that would be appropriate for rendering of the entire globe, to the 1m resolution needed to render a local neighborhood. Cartographers have long struggled with relationships between maps at different scales, but not over this large a range.

3) Smooth and convenient input and output, suitable for dynamic variation, so that it can be easily used for adding 3-D or multi-D data.

3.2.2 the selection of 2-D measure space and the definition of measurement

The original conception of map projection is just the transformation of curvilinear coordinate (B, L) on the 3-D ellipsoid to the plane (XOY). The main reason is decided by that the map carrier is a plane paper and plane measurement technique. Now another choice can be taken as long as the basic spatial characteristics can be maintained.

People can directly select the projection from the practical 3-D space (B, L, H) to (B, L) along the normal line as the space mathematic basis, and select the spheriodal earth geometry system decided by 2-D field (B, L) as the measure space. And its principle measure is solely decided by its positional coordinates according to the following definition.

The known values are point 1(B_1,L_1), point 2(B_2,L_2), flatten ratio α, the major axis a, the second axis b, the first excentricity e and the second excentricity e', the distance from point 1 to the point 2 is

$$S_{12} = K_1 b(\Delta\sigma - d\Delta\sigma) \qquad (1)$$

The direction angle from point 1 to point 2 is

$$A_{12} = \mathrm{arctg}(\cos u_2 \sin\Delta w/(\cos u_2 \sin u_2 \cos u_2 \cos\Delta w) \qquad (2)$$

$$\mathrm{tg}\, u_1 = (1-\alpha)\mathrm{tg}B_1, \quad \mathrm{tg}\, u_2 = (1-\alpha)\mathrm{tg}B_2, \quad d\Delta w = 0.$$

The following formulas are calculated by conditions in circulation.

$$\Delta w = L_2 - L_1 + d\Delta w$$

$$\mathrm{tg}\Delta\sigma = \frac{[(\cos u_2 \sin\Delta \mathrm{w})^2 + (\cos u_1 \sin u_2 - \sin u_1 \cos u_2 \cos\Delta \mathrm{w})^2]^{1/2}}{\sin u_1 \sin u_2 + \cos u_1 \cos u_2 \cos\Delta \mathrm{w}} \qquad (3)$$

$$\cos u_n = \cos u_1 \cos u_2 \sin\Delta w/\sin\Delta a$$

$$\cos 2\sigma_m = \cos\Delta\sigma - 2\sin u_1 \sin u_2/\sin^2 u_n$$

$$V = 1/4\,\alpha \sin^2 u_n$$

$$K_3 = V[1+\alpha+\alpha^2 - V(3+7\alpha-13V)]$$

$d\Delta w = (1-K_3)\alpha \cos u_n [\Delta\sigma + K_3 \sin\Delta\sigma (\cos 2\sigma_m - K_3 \cos\Delta\sigma \cos 4\sigma_m)]$

When the circulating calculation made the variation of $d\Delta w$ smaller than the demanded deviation, then the following formulas are calculated:

$t = 1/4e^2 \sin^2 u_n$,

$K_1 = 1+t\{1-t/4[3-t(5-11t)]\}$,

$K_2 = t\{1-t[2-t/8(37-94t)]\}$

Investigating the above distances, it is clear that in the set of {B, L} K, the distance S is a transformation which transforms {B, L}×{B, L} into the real number field R. For any points 1, 2, 3, there exists:

$S_{12} > 0$, only when $1 = 2$, $S_{12} = 0$

$S_{12} = S_{21}$,

$S_{12} + S_{23} \geqslant S_{13}$

Therefore, (B, L) is a measure space and distance S is a measurement scale.

The above 2D measure space is better than the plane rectangular coordinate space of any map projection and its measure distance can reach the required accuracy, while the directional conception is also more accurate than any plane rectangular coordinate space. Moreover, its area without any limitation and can be used for the whole world. For example, it is better than the Gauss projection, and there is no deviation of meridional convergence in its direction and there is not any error of 0.28% in the 6 degree zone.

The projection in the 3D space is very simple, and the measurement is just needed. It can be obtained from computer in real time.

Accordingly, we define the area of polygon on this projection, and thus a precise visualization measurement system on the whole world can be organized.

3. 3 **Model for practical application**

As a 2D field in the above {B, L} space, B, L can be taken as two axes and they are perpendicular to each other. Radian measure or degree can be used as units. Because the map sizes are different for different units, we may introduce a constant K into the formulas. So the practical model is as following:

$$X = K\cdot L$$
$$Y = K\cdot B \qquad (4)$$

where B, L take radian measure as unit. Obviously, if X, Y take meter as their unit length, then the practical physical meaning of K is the number of meters relative to one radian.

It is clear that constant K is related to the demanded resolution of image symbol, i.e., the scale, while the precision of B and L is related to the surveying or the primary precision. The resolution and precision of these two sides should be match with each other. Let K be the radius of mean curvature of the earth R, formula (4) can be used to discuss the problem of planar image expression with the real earth's spatial size. i.e.,

$$X = R\cdot L$$
$$Y = R\cdot B \qquad (5)$$

We can take it as the detailed expression of curvilinear coordinates on the plan.

3.4 Spatial property analysis of the practical model

3.4.1 Discussion on the geometrical characteristics

Formula (5) is equal to the spherical equidistant tangential cylindrical projection with R as the radius of mean curvature. However, the practical concepts are different. The former is the planar image expression of the geodetic coordinate system (B, L), and it is a 2D measure space with enough ellipsoidal geometrical measurement precision for the whole world. Here, the 2D image expression is apart from the measurement, but they are closely united each other, while the latter is a 2D planar expression of a map projection and a planar measurement system. It takes the measure of 2D Euclidean spatial distance. This is just equal to the 2D planar paper and ruler, compasses, protractor and other measure work cases. And the former does not have nor can use the measure work as the latter. There exist deformations in length, angle and area controlled in a definite range. But in the former situation, there is not any notable deformation in the whole world area, and it keeps good measurement properties.

It may be said that the coordinates of the model is of those as the equidistant tangential cylindrical projection's, and its measurement is of the geometrical system of the ellipsoid. It is obvious that the expression on the screen is a kind of relative partial distribution. It is impossible to take planar measurement. Therefore, from the view of conception, it is a "map projection", but not an ordinary projection.

In the past, a map projection represents a 2D planar expression and planar measurement. These two are united. When the expression can not meet the need of measurement, extra notes are added. This is a historical perplexity of mapping. However, these two are divided on the computer. The expression represents the geometrical and topological characteristics of an object, and it also keeps the spatial characteristics, while the measurement can precisely state the situation of spatial measuring values.

3.4.2 Discussion on the topological properties

The geographical layers on the earth surface form a solid space. Theoretically, the topological properties of 3D and 2D are different. But we postulate more or less that the geographical layers and zones are topological homeomorphism with an open sphere, i.e., homeomorphism with an ellipsoidal surface which is deleted a point, or it can be looked as a homeomorphism with a plane.

Thus, theoretically, the adopted model and the applied model can keep the topological property of the earth surface or the information world of the earth. And the map projection also can keep the ordinary topological property.

3.4.3 Discussion on the definite resolution and geometrical errors

The situations as discussed above are theoretically precise and compact. However, the practical geographical information data are discrete under definite resolution and precision. Therefore, there exist two conceptions of geometrical and topological resolutions. In other words, the errors will affect the identity between geometry and topology. And the bigger the error is, the stronger the affection will be. It is evident that in the Gauss projection the seam produced between neighbouring projection zones may be seen as the topological nun-identity caused by the error produced by the projection, at the same time, the directional wrong impression caused by some bigger meridional convergence may be seen as the variation of geometrical characteristics.

However, the models (4),(5) mentioned above are perfect theoretically and practically in comparison with ordinary projections. Thus, The space and the projection selected by us can strictly and reliably guarantee the spatial geometrical and topological characteristics of the tremendous geographical space and even of the whole world geographical entities, And it is also suitable for the unified models of multi-resolution or even unclassified resolution.

3.5 The problem of data accommodation and output

The data source of the products of large GIS and Digital Earth is very wide and complicated. Mainly it can be generalized into the following three cases.

1) Several ellipsoids in different periods. The reduction methods of several coordinate systems into a united or special ellipsoid have been clearly solved in many literatures, and there are not many problems left.

2) The vector data in vector mode or partial vector data in combined mode are generally organized in the form of discrete geometrical data and topological data. Such as DLG and DEM can be included in this species. In this situation, only geometrical data should be transformed, no matter it is in the form of geodetic coordinates or map projection. The transformation of them can be fulfilled by use of ellipsoidal projection or map projection, point by point. The various analytical and numerical methods can be found in related references [4], [5], [6].

3) For the digital products, the transformation of figures, images, DRG, DOR and other source materials is different from that of the data of discrete point. The demands for their transformation is that the geometrical coordinates of their whole pixels are correct and keep the continuity of figures, images, DRG and DOR. Among the sources DRG can be the representative, especially the nationwide database of 1:250000 has been constructed, and the DRG of 1:50000 and 1:10000 even more larger scale (some part of the territory) is being developed. In view of this situation, the difference of points in the whole map should not be too large. Otherwise, the quality of the images would be evidently lower.

Therefore model (5) should be partly reconstructed to satisfy the above demands, i.e.,

$$\begin{aligned} X &= R_1 \cdot \lambda \\ Y &= R_1 \cdot \phi \\ R_1 &= \cos(P_m) \cdot R \qquad (6) \end{aligned}$$

Where, R_1 is the defined mean radius of curvature of the area. P_m is the radius of the relevant latitude. Such as for the area of whole China its territory ranges from 0^o to 55^o in north semi-sphere. Let R_1 be the radius of curvature at the latitude 38, i.e., 0.788R. Thus in the area of whole China the images of DRG product at the scale 1:50000 the maximum ratio of transverse condensation is 1/5 only, and the vertical variation is very small, and it would not affect the quality of the primary DRG in Gauss projection.

The Wanxiang Map Transformation Software can be used for DRG transformation according to formula (6), Which is a general transformation tool based on the broad sense numerical map projection transformation method. The application of this method in past several years proves that it is stable and reliable.

For the transformation of general used maps at scale 1:10000 to 1:1000000, the suggested control points for each map are listed as Tab.1.

For DEM interpolation in the model of formula (6), the distance is naturally to use the longitudinal and latitudinal differences. General speaking, for 1:50000 map sheet we take 0.5"or 1" interval and the transformation is seamless no matter how large the area is. Be similar to DEM, the transformation can be processed in the same way for DOM,DLG.

Tab.1 The suggested density of control points for partial transformation

scale	map sheet		density of cont. point		no. of c.p.
	long. diff.	lat. diff.	long. diff.	lat. diff.	
1:10000	3' 45"	2' 30"	3' 45"	2' 30"	4
1:25000	7' 30"	5'	7' 30"	5'	4
1:50000	15'	10'	7' 30"	5'	9
1:100000	30'	20'	15'	10'	9
1:250000	1° 30'	1°	15'	10'	49
1:500000	3°	2°	30'	20'	49
1:1000000	6°	4°	1°	40'	49

3.6 The advantages of the model

1) The coordinate system of GIS in the model uses the united geodetic coordinate system. Its structure is simple, rigorous and keeps precise measurement and preserves the geometric and topological characteristics.

2) It keeps continuing visualization in a very large area even in the whole world, and is suitable for different resolution and scales.

3) It is suitable for various data sources, outside systems, geographical data in different mode and dynamic situation. Its spatial data can be either geodetic coordinates that may directly be used, or rectangular coordinates in a known map projection which can be inversely transformed into geodetic coordinates. Furthermore, in case that some control points are known, the planar coordinates can be transformed by using the transformation method in broad sense and related software to solve the input problem.

Therefore, the model can connect smoothly the different static and dynamic data body and data sources and various applications.

4 Experiments of large scale GIS on the spatial mathematical basis

On the basis of the mentioned above spatial mathematical model, since 1995, we have been undertaking the making of DRG, DEM and the collection of vector and raster data in multi-resolution. And Wanxiang GIS software has been developed. In the mean time, industrial experiments of practical GIS have been made.

1) We have accomplished the water supply information system of Wuhan city. In which more than 3300 sheets of topomaps at scales of 1:250000, 1:50000, 1:10000, 1:1000(partial) and 1:500 and the information data of water supply installations were collected.

2) The border guard information system for Yunnan armed police.

3) The flood prevention and drought-resistance information system for Xiaogan City.

4) The state ocean information system.

5 Conclusion

The practice shows that a specialized GIS system for the vector data needed by DRG and DEM for a large area up to the whole country even the whole world can be constructed quickly based on the model. The continuing visualization of 2D and 3D in multi-resolution in the whole area can be realized just as in the real place, as well as the query and analysis of various informations. This combination of vector and raster data for GIS has been a technical trend for international advanced GIS. And the spatial mathematical basis put forward in this research is the important basis and the premise of success.

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