

# THE CLASSIFICATION OF PROJECTIONS OF IRREGULARLY-SHAPED CELESTIAL BODIES

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## ABSTRACT

For traditional maps we use regular reference surfaces as mathematical surfaces, for example, sphere, ellipsoid or tri-axial ellipsoid and also various classes of projections.

As the basis for development of projections of celestial bodies with irregular shapes the traditional cartographical projections have used. As the reference surface the regular surface of a sphere (being most simple for calculation) is used, and the altitudes of points on the irregular surface are measured from the reference surface towards normal. In this case, the points will have the same latitudes and longitudes as on a regular surface.

The classification of traditional projections using *a type of orthogonal cartographical grid* is most convenient and simple.

At this stage of development of irregularly-shaped celestial body's projections it is possible to classify them as follows:

- projections classified by shape of body,
- projections classified by area of mapping region.

*By shape of body* the projections subdivided into:

- projections of celestial bodies with shape that is very different from sphere, ellipsoid, tri-axial ellipsoid – the bodies with shape like potato – “Potatographic projections”;
- projections of celestial bodies with doubled shape;
- projections of celestial bodies with polyhedral shape.

*By area of mapping region* the projections subdivided into:

- planetary scale projections;
- projections allowing to map a body up to one hemisphere;
- region scale projections

## 1. INTRODUCTION

It is known that cartographic projections are determined in different ways. The general method is that the plane coordinates and differential scales of projections are expressed as functions of the geographical coordinates. Therefore as the basis for development of projections of celestial bodies with irregular shapes the traditional cartographical projections have used in a modified way.

As the reference surface the regular surface of a sphere (being most simple for calculation) is used, and the altitudes of points on the irregular surface are measured from the reference surface towards normal. In this case, the points will have the same latitudes and longitudes as on a regular surface [1].

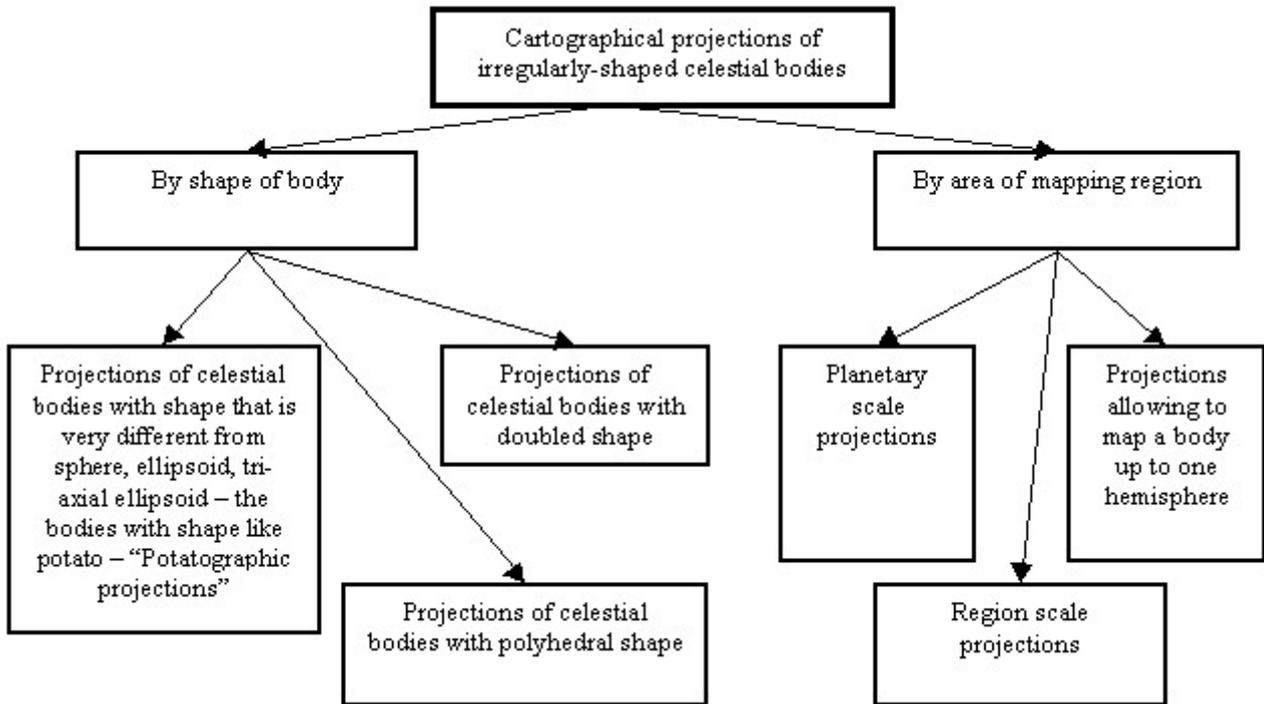
The radii are calculated from the center of the sphere towards the normal. For calculation of the data for projections it is necessary to know radius  $R_i=R+h$  which is represented as matrix with a certain interval on latitude from  $-90^\circ$  to  $+90^\circ$  and longitude from  $0^\circ$  to  $360^\circ$

In this case the common equations will be:

$$\begin{aligned} X &= f_1(\varphi, \lambda, R_i); \\ Y &= f_2(\varphi, \lambda, R_i). \end{aligned} \tag{1}$$

At the first step of development of cartographic projections of irregularly-shaped celestial bodies their classification can be shown as following scheme (see Tab. 1):

Table 1.



*By shape of body* the projections are subdivided into:

- projections of celestial bodies with shape that is very different from sphere, ellipsoid, tri-axial ellipsoid – the bodies with shape like potato – “Potatographics projections” [3] (Fig. 1);
- projections of celestial bodies with doubled shape (Fig. 2);
- projections of celestial bodies with polyhedral shape (Fig. 3)

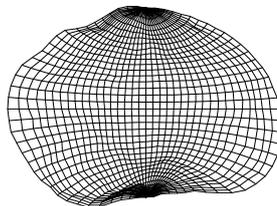


Figure 1. Cartographical grid of western hemisphere of Phobos in Postel projection (M. V. Nyrtsov, 1999)

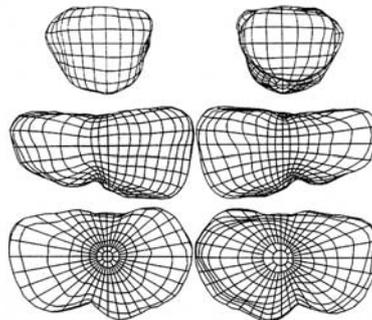


Figure 2. Cartographical grids of six orthographic views of asteroid 4769 Castalia [3] (P. J. Stooke 1998)

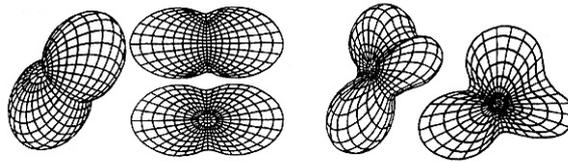


Figure 3. Cartographic grids of doubled and multilobate shaped celestial bodies in orthographic and morphographic projections [3] (P. J. Stooke 1998)

It is necessary to define the way of development of these cartographical grids more exactly.

The mathematical cartography considers mapping of simply connected objects with different shape. The calculation of projections of this kind can be carried out by two ways:

- The projections for doubled bodies can be calculated considering them as a simply connected area from one point of view.
- It is possible to use integrated projections similar to Kavraiskiy or Good projections.

By area of mapping region the projections subdivided to:

- planetary scale projections;
- projections allow to map a body up to hemisphere;
- region scale projections.

According to the last classification group we shall consider these classes of projections of irregularly-shaped celestial bodies on an example of Phobos – satellite of Mars. It is mapped globally; except regions of body up to hemisphere planetary projections are used.

## 2. PLANETARY SCALE CARTOGRAPHIC PROJECTIONS FOR GLOBAL MAPPING OF IRREGULARLY-SHAPED CELESTIAL BODIES

It is impossible to obtain a global view (without interruptions) of a celestial body in a single point perspective image. Therefore it is necessary to combine multiple images to show the whole surface of a body, and this requires the use of cartographic projections. The mapping of a body on planetary scale is very important in order to display significant knowledge about geology, geomorphology, albedo etc. [2]

Recent electronic or web atlases as well as traditional atlases have used for global mapping the equidistant along meridians square cylindrical projection and conformal triaxial ellipsoid projection. Before 1993 maps of celestial bodies on a planetary scale have been created using conventional cartographical projections without taking into account the irregular surface of a body:

- Phobos (Mars satellite) map in Mercator projection (Duxbury, 1974) (4,5),
- Map of Deimos (Mars satellite) part in Mercator projection (Noland, Veverka, 1977),
- Phobos (Mars satellite) map in normal conformal projection of triaxial ellipsoid (Bugaevsky, 1988)
- Map of Proteus (Neptune satellite) (Croft, 1992), the nucleus of comet Halley in square cylindrical projection etc.

The problem of developing planetary scale map projections for celestial bodies taking into account the irregular surface was formed in MIIGAiK in 1993. Then the investigation in this direction was started.

Some American scientists have preferred to use maps of opposite hemispheres of a body to represent the whole surface. Three mutually orthogonal pairs of views (north and south polar regions, and hemispheres with axial meridians  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ ), or four views from vertices of a tetrahedron illustrate the morphology of a body completely (e.g. Greeley and Batson 1997). For this reason the orthographic projection was recommended. But the orthographic view shows only one side of body at once and is insufficient to show the distribution of surface structures on planetary scale and determine their relationships. That proves the necessity for development of planetary projections. The MIIGAiK method offers mapping of the whole body including cartographic grids in a planetary scale and polar regions (north, south).

As an example of mapping we use Phobos, a satellite of Mars with mean radius  $R=13,5 \text{ km}$ . For calculation of planetary Phobos projections the digital shape determined by Peter Thomas (USA) with interval  $5^\circ$  was used. Therefore only the large features of relief were mapped.

The choice of projections depends on the purpose of the map to be created - to represent a body in a planetary scale and at the same time to show its physical surface. The traditional projections used earlier for the small-scale maps of the



If distance from point of view  $g$  to center of sphere  $O$  will be as  $D$  and distance to mapping plane as  $L$ , then for projections with negative mapping:

$$L_{\mp} = D + R, \quad (2)$$

for projections with positive mapping:

$$L_{\pm} = D - R \quad (3)$$

Depends of distance  $D$  perspective projections are divided to:

- Gnomonic  $D=0$
- Stereographic  $D=R$
- Orthographic  $D=\infty$
- External  $R < D < \infty$

The following projections are suggested to be the most useful for mapping the polar regions of irregularly-shaped celestial bodies: *The modified gnomonic projection, The modified stereographic projection, The modified orthographic projection.*

The following can also be recommended: *The modified orthogonal azimuthal projections - The modified Lambert projection, The modified Postel projection.* As an example the north and south polar regions of a body in a modified orthographic projection are illustrated (Fig. 6)

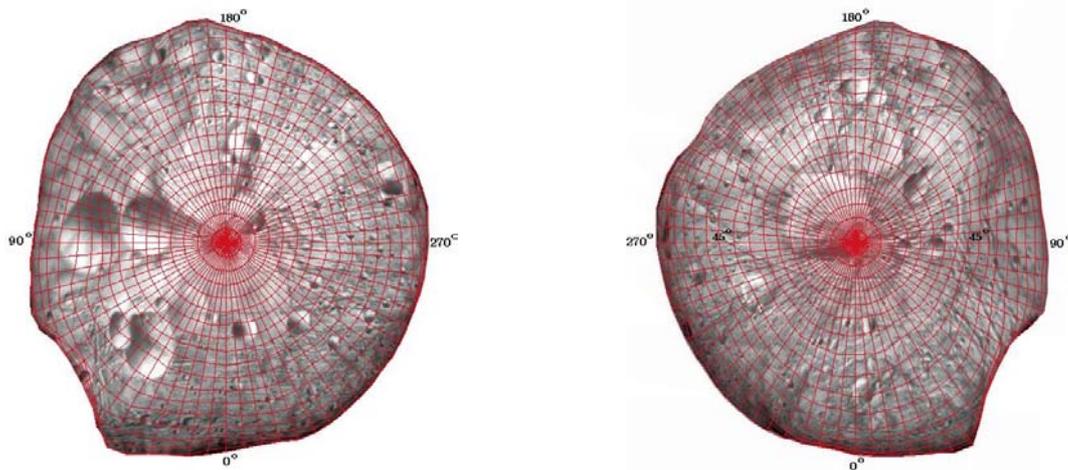


Figure 6. Modified orthographic projection

#### 4. CARTOGRAPHICAL PROJECTIONS FOR MAPPING OF WESTERN AND EASTERN HEMISPHERES

Sometimes planetary mapping with the addition of cartographical polar regions grids is not enough for complete visualization of a body. Commonly in planetary projections the shape of a map of a body is shown by the  $0^\circ$  and  $360^\circ$  meridians. The shape may look very different if different meridians form the rim. In this case mapping of a celestial body onto opposite hemispheres  $0^\circ, 180^\circ$  and  $90^\circ, 270^\circ$  is useful. This kind of mapping is most useful for bodies with non-symmetrical shape. It is possible to recommend for mapping of western and eastern hemispheres the following projections: *The modified transverse perspectively-azimuthal projections.* As an example the western and eastern hemispheres of a body in the modified orthographic projection are illustrated (Fig. 7)

#### 5. CARTOGRAPHICAL PROJECTIONS FOR MAPPING A REGION OF AN IRREGULARLY-SHAPED CELESTIAL BODY

With new spacecraft missions giving the possibility to get detailed data about irregularly-shaped bodies it will be necessary to make large-scale mapping. For this purpose it is possible to use various modified conventional projections applied for the Earth mapping as conic, cylindrical, and azimuthal and etc. Some projections described above can be used also for mapping of local regions of a celestial body, for example *The modified equidistant along meridians square cylindrical projection* – for mapping of equatorial areas, *The modified gnomonic projection* – for mapping of areas near the poles.

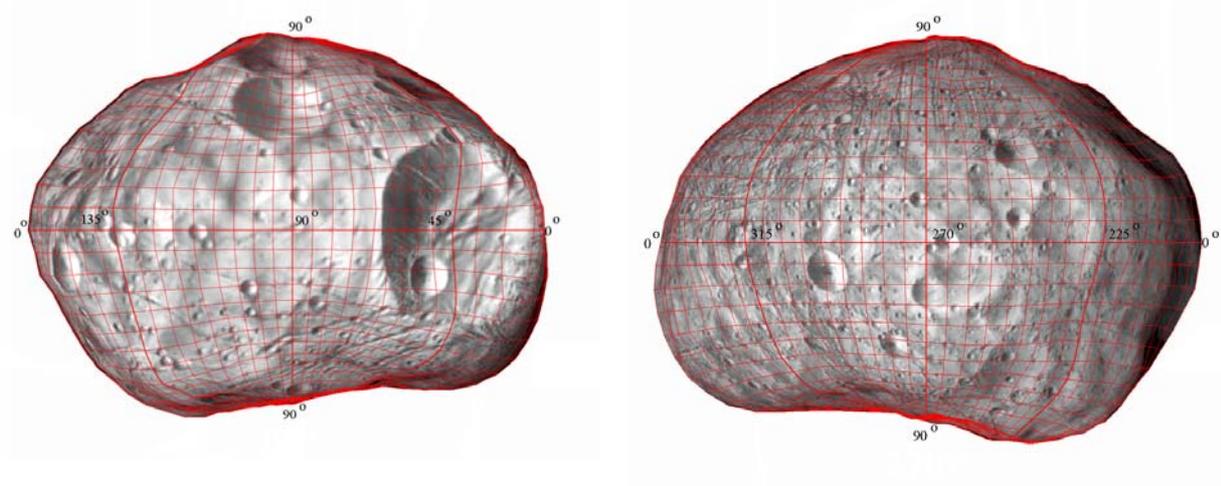


Figure 7. Modified orthographic projection

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## **Biography**

Maxim V. Nyrtsov was born in 1975. He graduated from the University for Geodesy and Cartography (MIIGAiK) in 1997. His major is engineer-cartographer. He is doing his PhD and nowadays he is teaching students at the Department of Graphics and Cartographical Design of the University.

In 1999 he participated in the 19<sup>th</sup> International Cartographic Conference in Ottawa, Canada, where he made a presentation on the topic “Mathematical planetary cartography basis for non-spherical celestial bodies.”

In 2001 he participated in the 20<sup>th</sup> International Cartographic Conference in Beijing, China, where he made a presentation on the topic “The problem for mapping irregularly-shaped celestial bodies.”

Fields of his interests: Mathematical Cartography, Planetary Cartography, GIS, Cartographic design.