

## **CHILEAN VELOCITY MODEL**

AUTHOR: LAUTARO RIVAS REVECO

DATE: AUGUST 12 2009

Because the Earth is a dynamic body, its shape changes constantly, so the reference system used to give coordinates to a point over the earth surface must have the capacity to adjust to this constantly changes.

The theory of tectonic plates and continental drift, are the phenomena that cause the movements of continents, earthquakes and volcanic explosions, which in large quantities cause changes in the Earth's surface.

The topography of the land isn't regular; there are large variations between the vertical ridges, valleys and sea trenches. That's because a new geometric figure called Geoid appears, which aims to be the best approximation of the real shape of the earth seen from space. Physically, the geoid is defined as the equal potential gravity surface at mean sea level. However, this is affected by the following natural phenomenon:

- Gravitational pull due to other celestial bodies in our solar system.
- Ocean currents.
- Ocean Freight.
- ice Melting.

These phenomena cause that the geoid is in constant change as they affect the configuration of water masses of our planet, which is the basis for the definition of the geoid.

The uneven distribution of continental masses, and the variable density of the materials that compose our planet, make that the geoid is not as regular as we would like, instead of that it present protuberance and depression, moving away from the real surface of the planet.

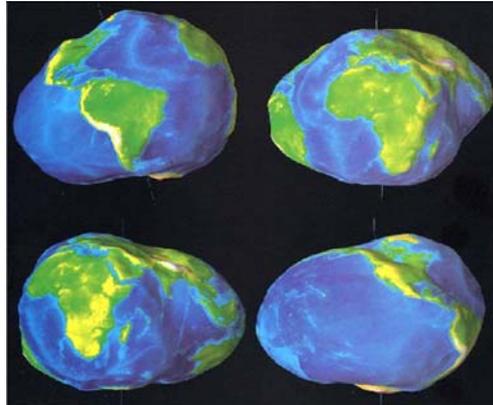


Figure No. 1 "Models Geoidales"  
 source: IGM 2007

Obtaining a reference surface with a mathematical definition for simple calculations is essential to allow the projection of elements over the land surface in order to create maps. The geoid surface can not be taken as reference, since, as we have pointed out, it's a very complex and irregular figure. That's why it is selected as reference figure an ellipsoid, but one that better matches the geoid and the earth surface.

Currently the ellipsoid used in our country is the GRS80 (Geoditic Reference System 1980), which is associated to a three-dimensional axis defined by ITRS (International Terrestrial Reference System), whose origin is the center of mass of the Earth. This ellipsoid is used worldwide in the definition of reference platforms.

Having the Ellipsoid it's origin at the center of mass of the Earth and, as we discussed, this is in constant change, we concluded that the ellipsoid is also dynamic.

All these factors cause that the coordinates determined for an element over the surface of the planet experiment translation and rotation in time.

Translation is the movement produce by the displacement of the origin of the Cartesian reference system.

Rotation is the change of coordinates due to the movement of the axes of the cartesian reference ellipsoid.

These differences in the coordinates of any point located on the surface, cause that these are valid only for the period of time in which the measurement take place

The effects are evident. First the coordinates determined at different times, independently of the high level of accuracy of the equipment used, may not be comparable or standardized. It is for this reason that the geodesic coordinates can not be

used as accurate control element in time. The second effect is the direct impact on the geodetic coordinates as they suffers loose of accuracy in time.

## **OBJECTIVE**

To address this research, was first established a general objective for the hole investigation, then were determined the specific objectives to achieve the general one.

### General Objective

Develop a velocity model for Chile; these is the creation of a digital program for the entire national territory, which allows entering any specific coordinate and as a result, obtain the coordinate referred to any interesting date.

### Specific Objectives

1. Analyze issues and technologies associated with the georeferenciation.
2. Identify the conceptual model of the velocity model for Chile.
3. Develop the velocity module through computational algorithms based on information from the RGN.
4. Develop the interpolation module according to mathematical models, based on the information of the velocity model.
5. Integrate both modules

## **METHODOLOGY**

The flow chosen to achieve the objective of this investigation was designed so as to fulfill the specifics objectives in sequence, focuses first on the velocities obtained from the data provided by the national geodetic network (RGN) and later in the development of algorithm allowing the calculation of velocities for the rest of the country.

The basic information to get the displacement velocities of the stations of the RGN, is a file generated by the IGM called "time series". The time series are files in which all the measurements of a single station are collected. This file is essential for the development of the model since it is here where you can see the differences in position that occur over time. These differences are obtained by calculating the differences of position between the first coordinate and the rest of them, in three dimensions: North, East and Height. Figure No. 2 shows a time series.

Época o fecha	Norte	Este	Altura
2000.0027	0.0000	0.0000	0.0000
2000.0055	0.0025	-0.0345	0.0086
2000.0082	0.0362	-0.0123	0.0040
2000.0110	0.0080	-0.0371	0.0117
2000.0192	0.0068	-0.0330	0.0084
2000.0219	0.0068	-0.0332	0.0079
2000.0246	0.0072	-0.0365	0.0111
2000.0274	0.0044	-0.0324	0.0077
2000.0301	0.0087	-0.0368	0.0113
2000.0329	0.0096	-0.0372	0.0101
2000.0356	0.0114	-0.0342	0.0098
2000.0383	0.0059	-0.0360	0.0108
2000.0411	0.0060	-0.0373	0.0123
2000.0465	0.0129	-0.0398	0.0154
2000.0493	0.0050	-0.0340	0.0089
2000.0520	0.0036	-0.0269	0.0036
2000.0548	0.0072	-0.0320	0.0092
2000.0575	0.0090	-0.0323	0.0079
2000.0602	0.0031	-0.0400	0.0116

Figure No. 2 Time Series of Santiago station. "  
Source: IGM

The information in this file can be seen more easily through a graphic, as shown in Figure No. 3. In it are shown the differences in the three dimension position (North, east and height) of one GPS station.

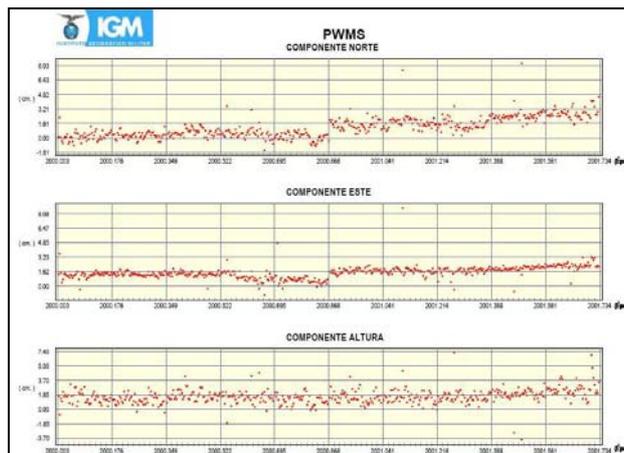


Figure No. 3 "Graphic of time series of Puerto Williams station"  
source: IGM

Once obtained the time series, the challenge lies on how to extract from them the three velocities in each station corresponding to the North, East and Height.

To obtain these values is necessary to find a mathematical function that better represents the behavior of the position along the independent variable, for this particular case, the time (date) that the position was obtained. In this sense, the least squares linear regression is the optimal way to achieve this objective.

This method consists on minimizing the differences between the data and the selected function, which for our case is a line; this is done developing the process described in the mathematical formula 1 using the information of time series:

$$\sum_{i=0}^n [f(x_i) - \phi(x_i)]^2 = 0 \quad (1)$$

With:

$\phi(x_i)$  = Function to use

$f(x_i)$  = Position in  $i$  Date

As the choose function is linear, the formula should be replaced as shown in 2.

$$\phi(x) = m * x + a \quad (2)$$

With:

m = Slope

A = Constant

Replacing 2 in 1 we have the equation 3:

$$\sum_{i=0}^n [f(x_i) - (m * x_i + a)]^2 = 0 \quad (3)$$

With:

$x_i$  = Date or time of measurement

$f(x_i)$  = Position differences en  $i$  date

This process should be repeated three times per station to obtain the constant and slope parameters for each of the lines (north, east and height). Once calculated, they

should be stored in a velocity file and then move to the next station. The whole process is done through computer applications developed especially for this purpose.

For the generation of velocities for the rest of the country where no data is collected, it is necessary to generate an interpolation of existing data from the previous step. To fulfill this, will be use the method of weighted average variable using the inverse of the distance.

This method uses as basis the fact that observations close between them tend to resemble more than those more remote, in other words, an item must have a behavior similar to the measurements or observations that are closer. It is natural to think that the contribution of an observation value interpolated using a weighted average, have a weight inversely proportional to the distance between the point and the observation.

The mathematical model for a weighted average is as follows according to equation 4:

$$\hat{Z}(x) = \sum_{i=1}^n \lambda_i * Z(x_i) \quad (4)$$

Whereas:

$\lambda_i$  = Weight of an observation

$Z(x_i)$  = Velocity of a GPS measure station.

n= Number of observations

As has been said, the method of calculating the weight for each observation is the inverse of distance squared, so the weights are determined according to the formula 5

$$\lambda_i = \frac{d_{ij}^{-2}}{\sum_{i=1}^n d_{ij}^{-2}} \quad \text{With} \quad \sum_{i=1}^n \lambda_i = 1 \quad (5)$$

Considering  $d_{ij}$  the distance between the unknown point and the location of the existing data.

Applying these algorithms to the velocity file, it is possible to calculate a velocity for any point within the national territory.

Finally join each of the modules described to function in an automated manner and through the Internet, which gave rise to the "velocity model for Chile." This allows to transport in time any position within the national territory, either geodetic or UTM coordinates in prompt and timely manner from any computer with an Internet connection.

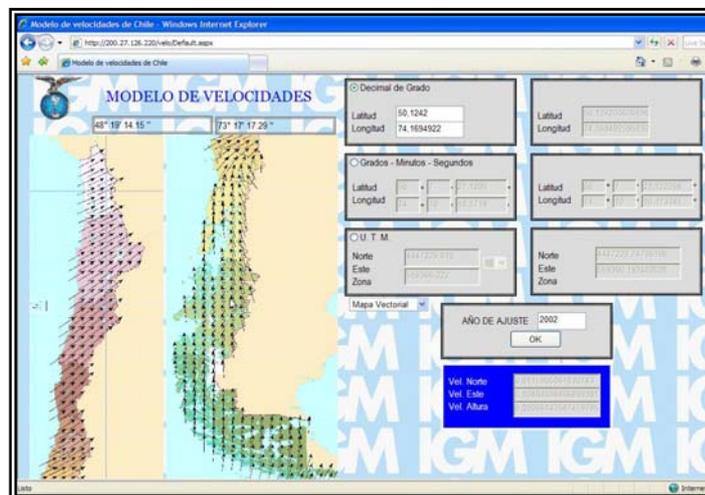


Figure No. 4 "velocity model for Chile."  
Source: IGM

## CONCLUSIONS

Some times when generating the time series of GPS stations, there are data that shows an unusual behavior, as shown in Figure No. 5. It is easy to see that in some time series ,certain positions are escaping the trend significantly. This error directly affects the calculation of the

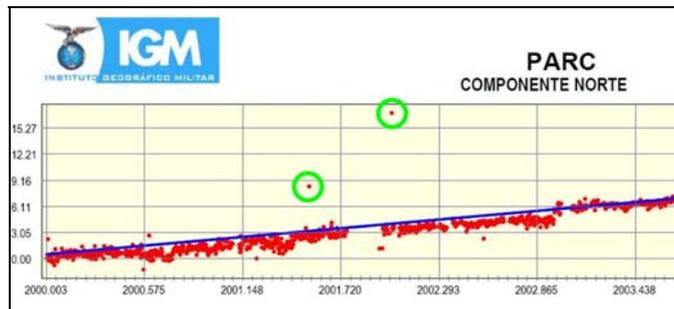


Figure No. 5 "Errors in Time Series."  
Source: IGM

Therefore it is necessary to analyze the time series when they are generated, with the aim of detecting these errors and, if necessary, refine the series before they generate new speeds. This allows the calculation of these rates is increasingly representative of the actual movement of the measuring station. The intention is to generate time series such as that shown in Figure 6 where the behavior of the differences in position shows a clear trend, with no data outside the normal range of measurement

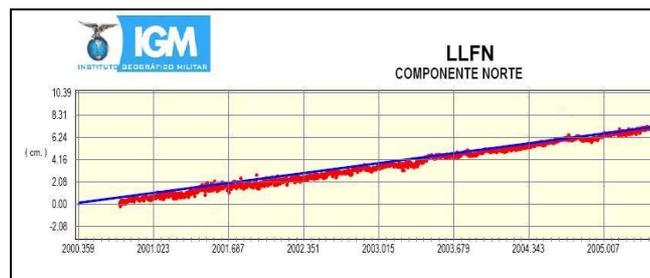


Figure No. 6 Series of pure time. "  
Source: IGM

Obtaining time series similar to the one of the figure, increases the accuracy with which the velocities are generated and as they are the basis for interpolation, also improves the accuracy of the model in general.

Another factor that influences the quality of the velocity model, is the distance that separates the data from the RGN.

The current velocity model was built for Chile based on information from 19 GPS stations. This means that the precision in executing interpolation is affected by this lack of density in the data.

According to the analysis during this project, the distance between data points and the calculated ones becomes the weights that give more importance to the closest velocities but those speeds imply an error that is generated by the standard deviation of the measurements and the process of calculating the speed itself. Greater distances, increase the incidence of the error in the result. This is why it is necessary

a greater densification of data by incorporating new measures that minimize the distance between the points.

The installation and maintenance of new permanent stations requires a high investment in equipment, personnel. A viable alternative is to seek cooperation with other institutions to enable the generation of new data, so as to refine the model.

In this context, during 2008 an agreement was conceived between the University of Chile and the Instituto Geográfico Militar. So during the years 2009-2010 will be installed 150 GPS measurement stations as shown in Figure No. 7. These stations are intended to form a seismic network measuring 24 hours seven days a week, sending the data through Internet. The IGM will be responsible for processing all the data generated by these measurement stations, becoming part of the RGN CHILE Sirgas.

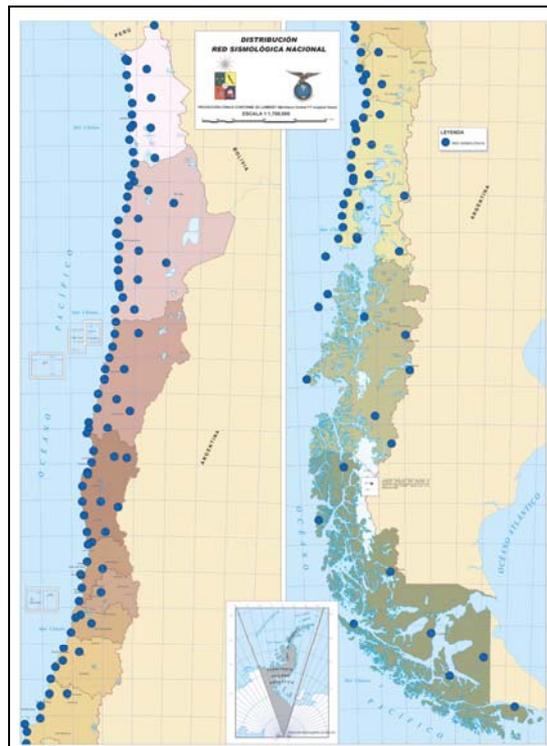


Figure No. 7 "National Seismic Network."  
Source: IGM

This will provide short-term with a considerable amount of active stations to generate velocities to increase the accuracy of the model.

One significant feature of the velocity model for Chile, is the ability to represent the movement of the national territory in a graphical, easy to understand way.

Finally it is important to say that the optimal function this model depends on the constant and regular update of the velocities that give rise to the interpolation of the data. This involves the daily processing stations belonging to the RGN who participate in this model.

## REFERENCES

Burrough, P.A. Principles of Geographical Information Systems for Land Resources Assessment. United States.

Seeber, Gunter. Satellite Geodesy. Berlin 2003.

Kiusalaas, Jaan. Numerical Method I Engineering with Matlab. The Pennsylvania State University.

Jekeli, Christopher. Geometric Reference Systems in Geodesy. Ohio State University, 2005.