ON THE ACCURACY OF DIGITAL ELEVATION MODELS AND THEIR GENERALIZATION

Abstract

The mathematical foundation in the generalization of digital elevation models (DEM) is revealed for the first time. It is pointed out that DEM generalization involves two steps. The first step is to interpolate an intermediate DEM which has minimal error at any DEM point and is isomorphic to the terrain so that topographic orderliness is preserved. The second step is to generalize the intermediate DEM to the final DEM. Generalization principal is outlined followed by a case study.

1. Introduction

Digital elevation models (DEM) is essential for various applications such as resource management, hydrological modeling, and military operations. Recent advancement in surveying and remote sensing technology has increased the availability of high-resolution DEMs (Waver and Lindenberger, 1999). Generalization is the process of abstracting a high-resolution DEM to a coarse-resolution DEM. For the purpose of this paper, we focus on model-oriented generalization (Muller et al. 1995) which aims to facilitate spatial analysis of the same phenomena at a higher level or smaller scale. This is different from graphics-oriented generalization whose primary interest is visualization.

Existing research on DEM generalization has mainly focused on Triangular Irregular Network (TIN) DEM which models terrain as a set of non-overlapping contiguous triangular facets. This paper will focus on grid DEM which samples the terrain at regular intervals. Compared to TIN which requires sophisticated data structure and large overhead of topology information, grid DEM is much more popular because of its compatibility with computer's array structure. However, automatic generalization of grid DEM is much more challenging in that points in a grid DEM can only be placed at designated locations. In reality, surface-specific points and lines such as ridges, valleys, peaks, pits, and saddles rarely fall exactly at these locations. If generalization is conducted mechanically by sampling the elevations of the designated locations only, critical terrain features which are very important for spatial analysis, will get lost. In order to preserve them, displacement of these points to some DEM points is necessary. This creates the other challenge of preserving their elevation accuracy. Essentially, when a 5-meter DEM is generalized to a 10-meter DEM, which in turn is generalized to a 20-meter DEM, then a 40-meter DEM etc., terrain is increasingly flattened out. Many important features will be eliminated or replaced by false alternatives.

2. Mathematical properties of terrain and DEM

As a model of the topographic surface, a DEM is expected to provide information on most, if not all, important terrain features with acceptable accuracy. To this end, the properties of the original terrain must be examined first. Mathematically speaking, terrain structure has three important aspects: (1) each point has a determined, though possibly unknown, elevation; (2) points are not independent but ordered which is best illustrated by the drainage patterns; (3) points of maximum or minimum elevations are of particular importance as they form surface-specific features such as ridges, valleys, peaks, pits, and saddles. These mathematical properties of terrain structure suggest that a high-quality DEM must have the following three characteristics:

a. Accurate point estimation, i.e. the error at each DEM point must be within tolerance. On this, numerous research has been conducted mostly from statistical perspective. Hu et al. (2009) presented approximation theory as a new theoretical framework and articulated the mathematics behind many empirical observations regarding DEM error. It is found that linear interpolation in 1D will result in minimum point error compared to other interpolation methods.

b. Isomorphism of DEM to the topographical surface in order to preserve topographic orderliness. This means if point a is higher than point b in the actual terrain, it must remain higher in the DEM and vice versa. Topographical orderliness is the reason behind the vertical patterns of many geographical phenomena such as temperature, vegetation, and soil. The isomorphism property is thus essential to assure the practical utility of a DEM. Despite its importance, the concept of DEM isomorphism was not articulated until Hu et al. (2009b). Through mathematical derivations, it can be shown that DEMs interpolated by linear interpolation can preserve topographical orderliness proactively.

c. Retention of surface-specific features such as ridges, valleys, peaks, pits, saddles etc. These features are formed by points of extreme elevations, they serve as terrain skeleton hence essential for spatial analysis. Theoretically, if a DEM can consist of all points in the terrain surface, have high point accuracy and is isomorphic to terrain, the preservation of all surface-specific features will be guaranteed. The challenge is that DEM is only a subset of terrain, hence the challenge is how to make a subset to inherit the property of the original set.

3. Theoretical framework

From the previous discussion, it can be seen that DEM generalization involves two steps: interpolation which results in an intermediate DEM of the desired spatial resolution, and generalization which processes the intermediate DEM so that critical terrain features will be retained. As discussed previously, the intermediate DEM resulted from the first step must possess two properties: minimum point error and isomorphism.

The total error at a DEM point is the sum of two components: propagation error which is due to the propagation of the errors in the source data to a DEM point and interpolation error which is caused by the imperfection of the interpolation function (Hu et al., 2009). In order to minimize the total error at a DEM point, the interpolation function must guarantee that the interpolation error is minimal. Moreover, the errors in the reference data should not be amplified during its propagation to a DEM point. Only if both of the two conditions are satisfied can the interpolation function results in minimum point error. On the other hand, minimum point error is not sufficient to produce an optimal intermediate DEM. Terrain is not a simple collection of points. Rather, it has an embedded orderliness where locations relate to each other according to their elevations. The topographic orderliness of terrain is best demonstrated by the drainage system where surface runoff flows systematically into tributaries and stream channels until it reaches the outlets eventually. A valid interpolation function should preserve this topographic orderliness, i.e. if point a is higher than point b, the interpolated elevation of a should remain higher and vice versa. Otherwise, the intermediate DEM is not of value hence it is pointless to proceed to generalization. Considering these requirements, linear interpolation in 1D is locked as the optimal interpolation function. Details on this interpolation method and how it results in minimum point error as well as preserving topographic orderliness can be found in Hu et al. (2009a , 2009b).

Once an optimal intermediate DEM is available, generalization is applied to create the final DEM. This step is the remaining challenge in DEM generalization research. Among the existing efforts, one approach is to use wavelets to separate a grid DEM into a component with coarse-resolution information and the other component containing the details. DEM generalization is then conducted by removing the details and retaining the coarse component only (BjØrke and Nilsen, 2003). Strictly speaking, this approach is simplification instead of generalization. It reduces the storage of a DEM but does not guarantee that critical terrain features will be retained.

Surface-specific points and lines are those that are irregularly distributed and adapt to the roughness of the terrain. Their inclusion in a DEM guarantees its structural fidelity Fowler and Little (1979) presented a method to extract surface-specific points and lines from a grid DEM. The extracted elements can then be ranked depending on their role in the local and global terrain structure. During generalization, these elements are moved to the nearest DEM point. In case of conflicts, low-rank elements always yield to high-rank elements. This strategy guarantees that all most important information will be retained using the finite number of points in a DEM.

4. Case study

The principals in DEM generalization discussed in Section 3 is applied to DEM generalization. For illustration purposes, we focus on DEMs interpolated from topographic maps. However, the principles outlined can be applied to other types of DEMs. The first step is to create a dense DEM. Assuming that the graphic resolution of a 1:50,000 topographic map is 0.1mm, the equivalent DEM then have a spatial resolution of five meters (figure 3). This 5-meter DEM is called the dense DEM in the sense that it contains the maximum number of DEM points allowed by the topographic map hence is expected to be a faithful model of the terrain. Suppose the task is to generalize the 5-meter DEM to a 10-meter DEM, the x and y coordinates of the designated DEM points must be located first. Linear interpolation is then applied to interpolate the elevation of these points. The result is the intermediate DEM which has minimum point error and preserves topographic orderliness (Hu et al., 2009a, 2009b).

To generalize the intermediate DEM to the final DEM, the Voronoi region of each point in the 10-meter DEM is delineated first. Voronoi region is employed to model the region of influence of each DEM point. Because of the grid structure of a DEM, the Voronoi region of a DEM point is a square and contains four points from the dense 5-meter DEM (Figure 1). If no surface-specific points are available in the region, the new DEM point takes the average of the four points from the original 5-meter DEM, i.e. point O is the average of point a, b, c, and d. In the case that a critical point is available, point O takes the elevation of this critical point, which is equivalent to shifting the critical point to point O. If multiple critical points are found, each point is moved to the nearest grid point. Low-rank points then yield to high-rank points. During the generalization process, some of the DEM points may be moved horizontally. However, the movement is never more than half of the spatial resolution. The distortion in the horizontal dimension is thus nearly 0. The scale in the vertical dimension remains 1:1, hence terrain skeleton is correctly described in the generalized DEM.

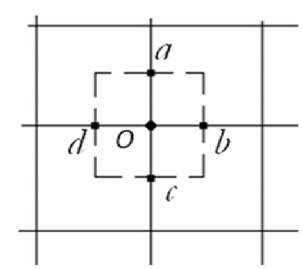

Figure 3 shows the results of the generalization process. The original 5-meter DEM is generalized to 10-meter DEM, 20-meter DEM, 40-meter DEM, and 80-meter DEM respectively. To evaluate the accuracy of these DEMs, contour lines were generated from each DEM and compared with the contour lines in the 1:50,000 topographic map. The figures show clearly that the contour lines from the generalized DEMs coincide well with original ones.

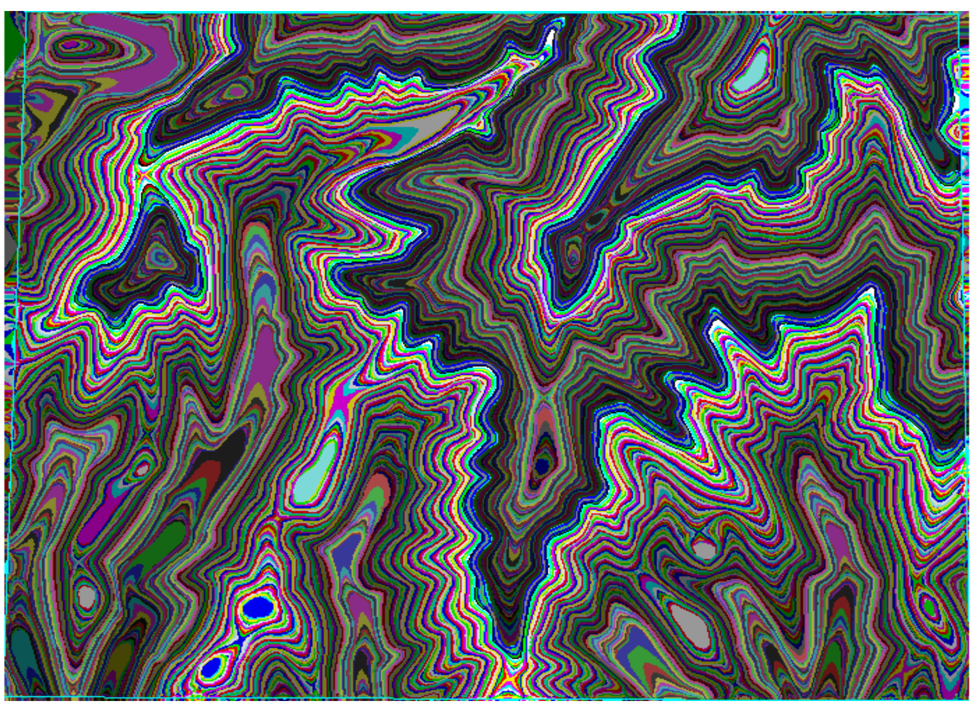

Figure 1. A 5-meter DEM interpolated from a 1:50,000 topographic map. This DEM has 695 by 559 pixels and preserves topographic orderliness.

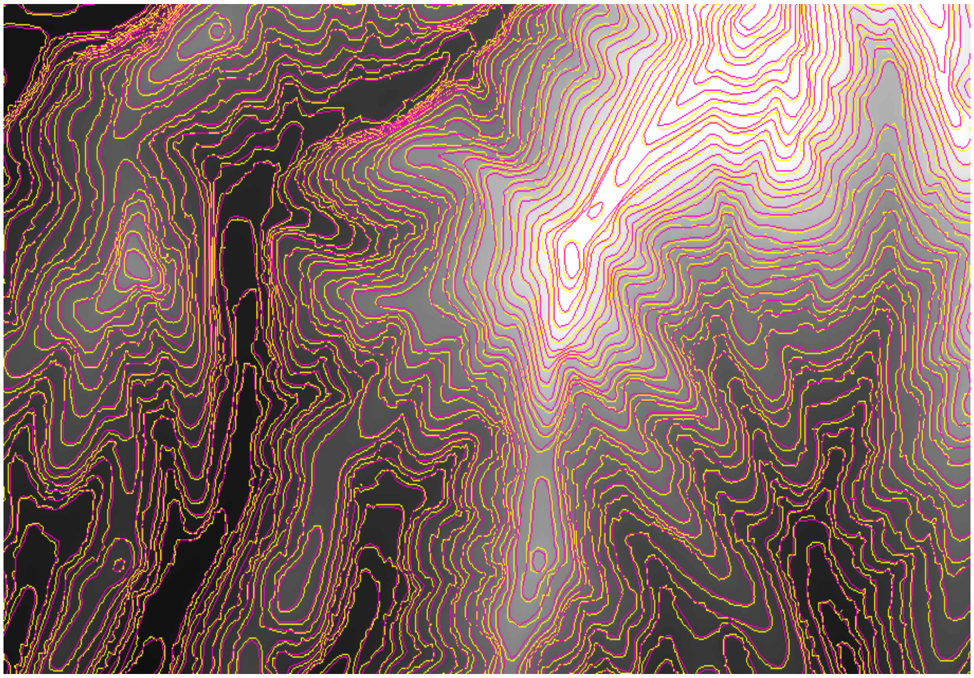

Figure 2. 10-meter DEM generalized from the 5-meter DEM (346 x 278). Contour lines were extracted form the 10-meter DEM and compared with the original contour lines. The extracted contour lines are marked by purple, and the original contour lines are marked by yellow. The contour interval is 10 meters.

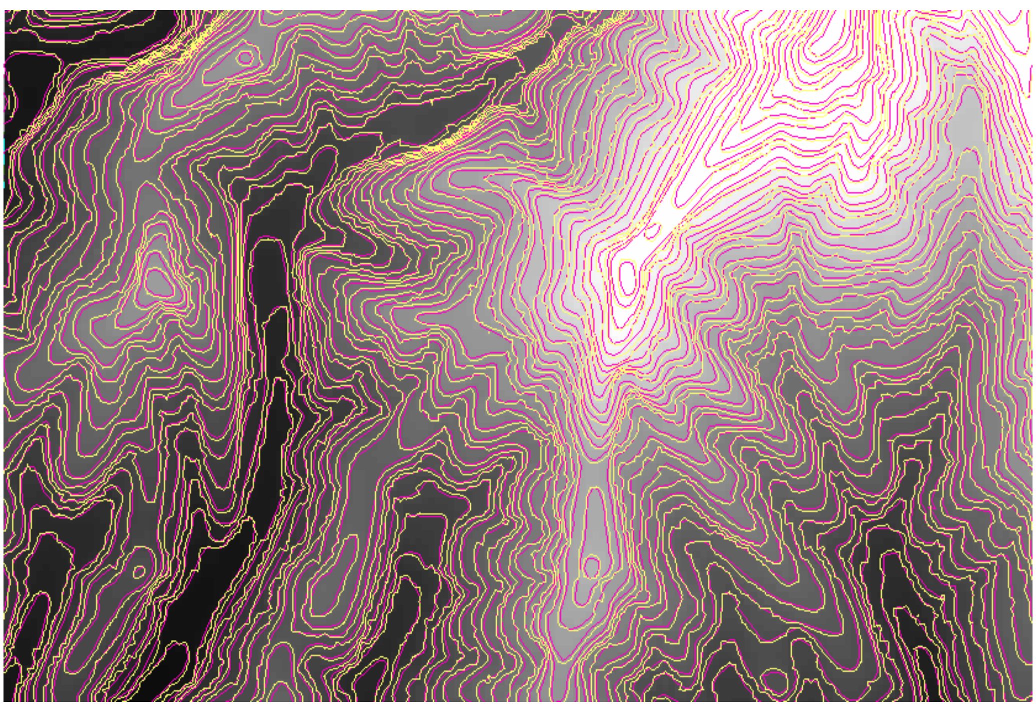

Figure 4c. 20-meter DEM of 173 * 139 pixels.

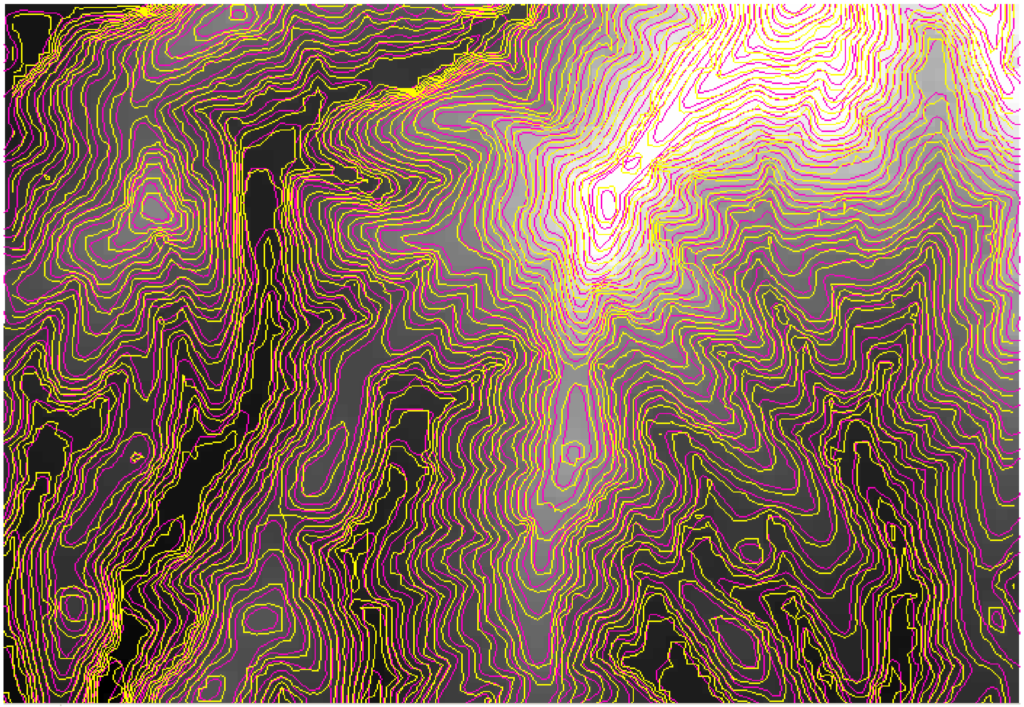

Figure 4d. 40-meter DEM of 85 by 68 pixels.

5. Conclusion

In this paper, a new algorithm on DEM generalization is presented. It is pointed out that DEM generalization involves two steps: (1) interpolation of an intermediate DEM which has minimum point error and preserves topographic orderliness; (2) generlization of the intermediate DEM through shifting and comparing surface-specific points. The case study presented illustrates the high accuracy of the algorithm.

Topographic map generalization has always been considered a challenge to the field of mapping and surveying. Many think the solution is NP-hard hence not available. Considering the connection between DEM and topographic maps, the algorithm presented int his paper opens up the door to solve the challenge of map generalization.

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