# A GOAL-ORIENTED EVALUATION OF LOCATION/ALLOCATION METHODS

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**Abstract.** The main purpose of any facility location is to select the optimal places that satisfy project goals. In location problems, the object is usually to optimize a function that is called objective function. The function defines the problem conditions and efficient decision parameters. Numerous methods are proposed to challenge the location facility issues. Each of them is appropriate for solving one type of problems according to its constraints.

In the framework of this research, facility location methods and problems are classified into four distinct categories based on their goals. Each class is then formulated as an optimization problem. In order to evaluate the efficiency of each method, a case study is considered. The methods are evaluated based on the data obtained from study area. The implementation results are compared quantitatively. In conclusion, application of each method was examined and presented based on the problem's goals.

**Keywords:** Facility location/allocation, Optimization, Classification, GIS, objective function

## 1 Introduction

Facility location selects the optimal location with maximum efficiency according to the goals of projects. Solving location problems needs to consider the effective constraints and criteria in its computational process [1]. Recent decades witness huge improvements in computational research and information technology, and as such the application of Geospatial Information System (GIS) in facility location is increased exponentially [2].

Spatial scientists are continuously working on mathematical modeling and solution of problems concerning the placement of facilities to minimize transportation costs, prevent placing hazardous materials near housing, outrun competitors' facilities, etc. Several objective functions relevant to each model, are also formulated [3]. In location problems, all input values such as demand rate, distances and time of service transition are considered as instant parameters and then outputs are estimated as decision variables in one point of time [4].

The systematic study on location problem formally began by Weber in 1909. He considered how to position a single warehouse to minimize total distance between it and several customers 5]. Since the mid 1960s, the study of location theory has been developed and basic problems in this context have been formulated as static and deterministic problems [6].

Static problems deal with constant known quantities as input. According to the problem constraints, the processing results give out a single solution to implement at one point in time. Following this, in 1964 Hakimi extended location theory with his researches to locate switching centers in a communications network in a highway [7].

Some of the problems require more facility centers to be located. Thus, multisource Weber problem has been defined [8]. The multisource Weber problem is defined as locating simultaneously m facilities in the Euclidean plane in order to minimize the total transportation cost for satisfying the demand of n users, each supplied from its closest facility [9].

Location problems can be solved through various strategies. Determining suitable solutions is difficult for users. Without a general solution classification, the examination and comparison of the methods maybe difficult if not impossible.

Accordingly, within the framework of this paper a general classification of location methods and their mathematical formulations are presented. The classifications are done based on the categories fitness functions and their applications in solving the problems . In order to compare the efficiency and the usage of each method, simulated data are generated. Models are classified into four categories. Software has been developed to test each category. The results are then compared quantitatively. At last, application scenarios that best suit each category are identified.

The rest of this paper is organized as follows: Section 2 describes how to classify the different location methods. Section 3 explains how to compare the methods by optimization problems. Finally, we give concluding remarks in section 4.

## 2 Classification and Mathematical Formulation

Facility location models are used in a wide variety of applications. These include, but are not limited to, locating warehouses, hazardous materials, public and private centers, rescue stations. Each of these location problems can generally solved on one of three basic continuous, discrete and network spaces. They all have different objective functions according to problem details. From a classification point of view, facility location models are different in their objective functions.

According to wide extent and diversity of problems that related to location and special characteristics of each location problem, several location methods have been presented for solving each of them. Apart from details of private constraints and conditions refers to each of different problems, almost all location methods can be formulated as a few general distinct classes according to their application and their applied fitness function.

Advantage of this general classification is in possibility of presenting a unique formulation for problems with similar constrains. Thus with existing differs in details, general prospect in method of solving problem is obtained. Therefore, applying constraints and conditions to the general formulae of the selected appropriate general method leads to the desired specific problem. This section provides classification and mathematical formulation of various location methods.

## 2.1 Median problem

First class is known as p-median class. P-median problem is a version of the multisource Weber problem [7]. The purpose of this problem is to find the location of p facilities to minimize total demand-weighted travel distance between demands and facilities [6].

For mathematical formulation of these problem decision variables defines as follow:

$$X_j = \begin{cases} 1 & \text{If it locates at potential facility site j} \\ 0 & \text{Otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{If demands at node i are served by a facility at node j} \\ 0 & \text{Otherwise} \end{cases}$$

In the above equations i is index of demand point and j is index of potential facility site. Accordingly, p-median problem can be formulated as (1) to (6).

$$Minimize \sum \sum h_i d_{ij} Y_{ij} \tag{1}$$

$$\sum_j X_j = 1 \tag{2}$$

$$\sum_j Y_{ij} = 1 \qquad \forall i \tag{3}$$

$$Y_{ij} - X_j \leq 0 \qquad \forall i, j \tag{4}$$

$$X_j \in \{0,1\} \qquad \forall j \tag{5}$$

$$Y_{ij} \in \{0,1\} \qquad \forall i, j \tag{6}$$

Where hi is demand at nod i, dij expresses the distance between demand node i and potential facility site j and p is number of facilities to be located.

As it can be seen in the aforementioned formula, the objective function (1) minimizes the total demand-weighted distance between facility centers and demand nods. The constraints (2) control that exactly p facilities be located. Equations (3) ensure that all demands refer to centers. The family constraint (4) allows assignment only to sites which facilities have been located. The family constraint (5) and (6) are binary requirements for the problem variables. With considering six above formulas together, solving this problem is leads to solve an optimization problem that needs to minimize (1) so that satisfy constraint (2) to (6).

### 2.2 Covering problems

A demand is said to be covered if it can be served within a specified time or distance [8]. Covering problems can be divided into two main parts. One is related to that in which coverage is required and another is related to that in which is optimized. The former is called covering problem and later is called maximal covering problem [13].

#### 2.2.1 Set covering problems

In the set covering problem, the objective is to select appropriate locations for facility centers so that minimize the cost of satisfying demands such that a specified level of coverage is obtained. In this problem coverage distance is fixed and the minimum number of facility centers hat is necessary for covering the all demands, is determined. By the set covering problem one can examine how many facilities are needed to guarantee a certain level of coverage to all users.

This problem can be formulated by equation systems (7) to (9).

$$Minimize \sum_{j} c_j X_j \tag{7}$$

$$\sum_{j \in N_i} X_j \geq 1 \qquad \forall i \tag{8}$$

$$X_j \in \{0,1\} \qquad \forall j \tag{9}$$

In above equations, cj is fixed cost of sitting facility at node j, S is maximum acceptable service distance or time and Ni is the set of facility sites j within acceptable distance of node i.

The cost of facility location is minimized by the objective function (7). The family constraint (8) explains that all demand i have at least one facility located within the acceptable service distance. Constraint (9) insures integrity for the decision variables. This formulation makes no different between demand nodes. All of the nodes are assumed that have an equal demand rate and must be covered within the specified distance.

#### 2.2.2 Maximum covering problem

Maximum covering problem seeks to maximize the amount of demand covered within the acceptable service distance by locating a fixed number of facilities. For mathematical formulation of this problem, decision variable Zi defines as follows:

$$Z_i = \begin{cases} 1 & \text{if node i is covered} \\ 0 & \text{Otherwise} \end{cases}$$

This problem can be formulated by the formulae (10) to (14).

$$Maximize \sum_{i} h_i Z_j \tag{10}$$

$$Z_i \leq \sum_{j \in N_i} X_j \qquad \forall i \tag{11}$$

$$\sum_j X_j \leq p \tag{12}$$

$$X_j \in \{0,1\} \qquad \forall j \tag{13}$$

$$Z_i \in \{0,1\} \qquad \forall i \tag{14}$$

The objective function (10) maximizes the amount of demand points covered. Inequalities (11) define which demand nodes are covered within the fixed distance covered. Constraint (12) controls the number of facility centers and limits it to fixed number. Similar to (9), the constraints (13) and (14) are integrity constraints for the decision variables.

The relationship between the p-median problem and maximum covering problem can be found by defining d'ij [14-15] as follows:

$$d'_{ij} = \begin{cases} 0 & if \quad d_{ij} \leq S \\ 1 & if \quad d_{ij} > S \end{cases}$$

Using d'ij in p-median equations, the amount of no served demands within coverage distance S will be minimized. This means to maximize the amount of demands served within S. With this transformation, maximum cover problem can be viewed as a spatial case of the p-median problem with an additional parameter S.

## 2.3 Center problems

The p-center problem in a mathematical framework is known as a MinMax problem. That means the aim is to minimize the maximum distance between any demands and is nearest facility. If D is maximum distance between a demand node and he nearest facility, p-center problem can be formulated as equation (15) with constraints (16) to (21).

The object function (15) minimizes the maximum distance between any demand node and its nearest facility. Constraints (16) to (18) are similar to constraints (2) to (4).equation (19) determines the maximum distance between any demand node and the nearest facility. Constraints (20) and (21) are integrality constraints for decision variables. In case of decision variables Yij could be fractional; one demand node might be served by multiple facilities.

$$Minimize \quad D \tag{15}$$

$$\sum_j X_j = p \tag{16}$$

$$\sum_j Y_{ij} = 1 \qquad \forall i \tag{17}$$

$$Y_{ij} - X_j \leq 0 \qquad \forall i, j \tag{18}$$

$$D \geq \sum_j d_{ij} Y_{ij} \qquad \forall i \tag{19}$$

$$X_j \in \{0,1\} \qquad \forall j \tag{20}$$

$$Y_{ij} \in \{0,1\} \qquad \forall i, j \tag{21}$$

## 3 Simulation

In order to compare the efficiency of classified methods in solving a unique problem, a section of central part of Tehran, the capital city of Iran, with nineteen hospitals has been chosen (figure 1).

The primary purpose is to find the best place for establishing a catering center to satisfy the existing hospitals demands. Demands rate has been assumed equal for all hospitals. Euclidean distance has been considered as the only efficient parameter in satisfying the demands. Covering distance, in cover problems, of 160 units has been assumed..

According to the previous equations the best places for establishing catering centers are determined and presented. (Figure 2). In figure 2, circles indicate facility centers and blue squares indicate hospitals as demand points that have been referred to related facility centers with lines.

Figure 1: A section of central part of Tehran, capital of Iran, as sample region

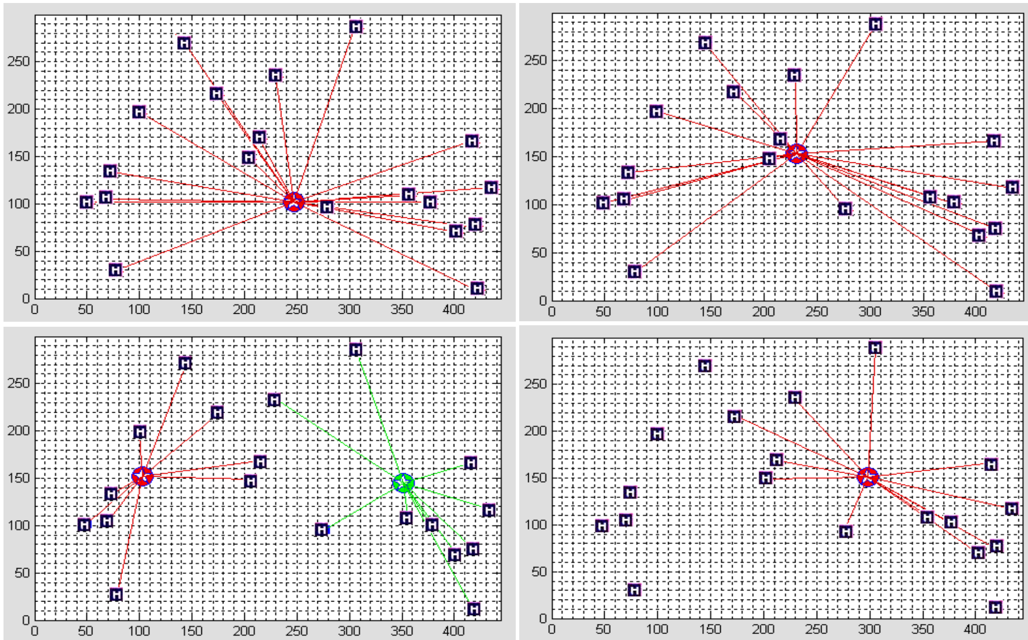


Figure 2: The best places for establishing facility centers from top and right respectively are median, center, max-cover and set-cover

Table 1 presents the results of numeric computations.

Table 1: Numeric result of comparing different location problems

| | Coordinates of selected facility centers | Total demand distance between facility centers and demand nodes | Maximum distance between any demand node and its nearest facility | Number of covered demand nodes |
|---|---|---|---|---|
| Median problem | *X1=230.413 , Y1=153.518* | 2745.14843 | 236.98601 | 19 |
| Center problem | *X1=246.564 , Y1=102.005* | 2871.12880 | 196.56422 | 19 |
| Max cover problem | *X1=298.448 , Y1=151.269* | 1321.59981 | *141.80786* | *12* |
| Set cover problem | *X1=103.057, Y1=151.799*<br><br>*X2=351.437 , Y2=144.416* | 1734.28714 | 151.23283 | 10<br><br>9 |

Table 1 presents the location of best facility centers using various methods. As it can be seen, the locations are not the same. Thus, each problem must be categorized properly, and then, solved using unique solutions presented in this paper. The solutions must be tuned according to the application at hand, its specific constraints and goals.

Figure 3 illustrates the position of facility centers on the city map.


Figure 3: Selected locations using each of four methods on sample region

## 3 Conclusions

This paper addressed the location problems. The problems were classified into four distinct categories. Each category is examined using simulated data, and the results are compared quantitatively.

We have demonstrated that in problems such as establishing public facility centers like school and hospital, p-center method is appropriate. Because in this type of problems the purpose is to satisfy all demand nodes. Even the discrete nodes that are located far away must be served similar to other demand nodes. In other words, facility centers must be located in a place that is close to demand nodes. Thus, p-center problem is more suitable in this type of problems.

If the problem is to establish emergency centers such as fire stations, we need to define a coverage distance or time for the services. Thus, the objective is to cover the region with minimum centers considering a fixed cover distance or time. In this type of problems, set cover is the best solution.

In problems such as determining the optimum location for establishing markets, p-median is an appropriate method. In such problems, although discrete demand points are far from centers but nearing to focused section from the viewpoint of demand nodes, is economical.

If the problem is related to establishing private facility centers such as restaurants, maximum cover can be the best choice for solving these problems. Because in establishing private facility centers, the goal is to serve the maximum demand nodes in a determinate cover distance.

We demonstrated that through the proposed classification, almost all location problems can be categorized.

As future researches, authors want to classify and determine mathematical methods for solving dynamic location problems.

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