CO-398

USING ENTROPY TO ASSESS EFFICIENCY OF TERRAIN REPRESENTATION

FAIRBAIRN D.
Newcastle University, NEWCASTLE UPON TYNE, UNITED KINGDOM

INTRODUCTION

There are a number of methods for representing terrain on a standard topographic map, which can be used singularly or in combination. The project described in this paper addresses whether it is possible to identify the most efficient method of portraying such terrain data. The aim is to determine how much information is communicated from the map to the user through the symbolisation and design of graphical elements which represent the topographic surface. The symbols which are used reflect locational characteristics, arrangements, structures, patterns and topologies of an area, and associated meaning and attributes of the terrain data.

The primary characteristics of terrain – elevation, slope, aspect and surface appearance – are denoted by such cartographic symbolisation, either directly, by implication, or as a result of interpretation or calculation based on the representation. Thus a map comprising spot heights alone directly communicates position and elevation of identifiable singular points. However, knowledge of the slope at a singular point relies on an informed analysis of the array of spot heights around it. Figure 1a shows an area of terrain represented by a spot height map – the information presented is sufficient for direct identification of points and their elevation. Figure 1b enhances this array of points by creating a triangular irregular network constructed using a Delaunay triangulation of the spot height array.

Does Figure 1b display any more information that Figure 1a? The data portrayed is identical, but the addition of the network implies a surface coverage of triangles, which can be used by the map viewer to more confidently appreciate the terrain facets, their slope and aspect. Such a TIN structure can be used by software to further derive contour patterns, create hillshading, and perform volume calculations.

If the efficiency of terrain maps can be defined as the successful portrayal and dissemination of information, it should be possible to determine which of these two representations is the more efficient, as a cartographic product.

There are several other methods of terrain representation, shown in Figure 2 (covering the same area as Figure 1). Some are based solely on point symbols (for example, a regular array of spot heights); some on lines (contours at variable intervals are shown in Figures 2a and 2b); 3D strings or ‘gerippe’ lines highlighting terrain structures such as ridges and valleys; and others are continuous over the entire surface (hypsometric tints, for example in Figure 2c; landscape facets, each with constant slope and aspect; and continuously varying hill shading – Figure 2d). The information content of these representations can be determined to comparatively assess their efficiency.

MEASURING INFORMATION CONTENT

Attempts to measure the information content of map representations can use indices such as entropy (Bjorke, 1996). Entropy calculations are usually based on probability theory applied to the presence, absence and frequency of occurrence of symbols. This simplified measure of the information content of a map image draws from classical entropy calculations initiated by electrical engineers in the 1950s who were interested in the information disseminated in pulses (recorded as bits) by a digital signal in telecommunications. Such information content measurement is complicated in maps by a range of inherent and added features: unlike one-dimensional communications systems, maps possess scale and design (possibly with associated emotional input), and they also use symbols as representative of feature attributes, measurements and properties. Further, they are used for specific purposes (e.g. navigation), and may well be interpreted within specific contexts (knowledge, or lack of knowledge, of an area).

These aspects, along with (for terrain data), the range of different representation methods employed, means that differing types of entropy measurement can be identified, dependent on the type of symbol used (e.g. points, lines, areas, grids, surfaces, images). Further modifications of entropy calculation to consider the presence of map symbols, their attributes, the values they convey, and their distribution, can be applied (Li and Huang, 2002).

Scientifically, entropy is equated to ‘uncertainty’ (Feng and Wang, 2009), and, simplistically, if uncertainty is reduced then information conveyed is maximised. In fact, what is maximised is additional information beyond that which is predictable or statistically likely. Thus, low values of entropy reveal a more ordered state, one in which the transmission of ‘new’ information is optimised.

From a mathematical perspective, calculation of entropy is based on probability such that entropy equals minus the sum of the probabilities (or relative frequencies) of each symbol multiplied by the logarithm of that probability:

$$H(S) = -\sum_{i=1}^{n} p_i \ln p_i$$

where H is the entropy of a source S, and p is the discrete probability of each element i in the source.

A major difficulty is that in entropy calculation, the centrality of the concept of probability of a symbol's occurrence is compromised by the different types of symbols used on maps, and the different methods of assessing probability. In varying contexts, the terms 'geometric entropy', 'positional entropy', 'attribute entropy', 'fractal entropy', 'grid entropy', 'pixel-based entropy', 'image entropy', and others, have been used in practical tests designed to measure entropy. And if redundancy of data presentation methods are presented (for example, the common graphical combination of contours and hillshading to represent relief), then it becomes more difficult to identify an accurate measure for such 'joint entropy' (Wang et al., 2010). Entropy is additive if the two methods are completely statistically independent, but the clear relationship between the hillshading image and the contour frequency and distribution would require some adjustment to be made in the calculation of joint entropy.

The aim of this project, presented here at an interim stage, is to answer the questions: can we identify the type of entropy applicable to each terrain representation symbol type and quantify it sufficiently to determine which gives the optimal information portrayal? would a combination of ALL of these methods yield more information than a single parameter display? and to what extent is such a quantification of the information portrayed on the map valid?

APPLYING ENTROPY MEASUREMENT TO TERRAIN DATA

Initial attempts have been made to assess the utility of entropy as a measure of information content for lines and for images. The sensitivity of such measures has also been considered by measuring entropy of representations which differ in appearance and level of resolution despite covering the same terrain area and being sourced from the same dataset.

To answer the first question posed above, the entropy value calculated for the distribution of point symbols identifying notable points on the terrain surface, and displayed in Figure 1a is 5.9726, whilst the addition of a triangulated network based on these vertices (Figure 1b) yields an entropy value of 6.6953. In both cases, the entropy was calculated using an area-based calculation, the former being the entropy of the Voronoi polygons (Figure 1c) which define each point's 'area of influence' (the method recommended by Li and Huang (2002) for use with point arrays), and the latter being the entropy value of the triangle polygons themselves. In order to normalise the entropy calculation (there are only 977 Voronoi polygons, but there are 2023 triangles, and adjustment must be made for mismatch of such elements) the calculated figures are further divided by ln(n) to reveal some agreement between the Voronoi polygons (0.8674) and the triangles (0.8796).

The normalised entropy value of a mapped square grid array of points, typical of sources of digital terrain model data, will always be 1 – the distribution is perfectly predictable, so there is no 'extra' information being presented by the map (Mendocino and Sole, 1997).

In the calculations thus far, whilst the distribution and number of point features and triangles based on these points as vertices have been taken into account, there has been no consideration of the actual elevation values of the points. Some form of 'attribute entropy' should be considered if we are to consider that the entropy measures do actually reveal information about the terrain surface.

Derivatives of the grid based data covering the same area as the TIN based DTM shown in Figure 1 is displayed in the individual maps in Figure 2. The calculation of 'image entropy', in which each map is considered as a continuous array of pixels (in this case grayscale values), the varying probabilities of which contribute to the index, has been undertaken for Figure 2c. The hypsometric tints shown here have an entropy value of 2.7338. It is noteworthy that the calculation of the entropy of the 50m interval contour pattern in Figure 2b (which, in effect, forms the outlines of the 50m hypsometric tint areas) yields a similar value of 2.6466. It is suggested that whilst the entropy calculation for the hypsometric tints is based on pixels and their frequency, and the entropy for the contour pattern is based on a probability measure based on their lengths, what is being measured here is related to the identical terrain surface represented in two different forms.

As shown by Bjorke (1996), a smaller contour interval (i.e. more frequent contours across an image) will yield a higher entropy value (for Figure 2a with a contour interval of 10m, entropy across the map is 4.2179, higher than for Figure 2b). Further, the more generalised contours shown in Figure 1d, also with a 10m interval, but derived from the TIN, have a lower entropy value of 3.953.

The easy automation of contour creation and ease and familiarity of use mean that contour maps are regarded as the most efficient method relief representation. But the range of other methods includes hillshading, a similarly easy-to-produce output layer from a digital terrain model. The continuous surface displayed by hillshading gives a potentially higher entropy value than contour lines. Image entropy has been calculated for the more varied grayscale picture of the terrain shown in Figure 2d has an entropy value of 4.8176.

CONCLUSION

This study is not yet complete. The interpretation of the measures obtained from these terrain data sets still needs careful consideration. The entropy index, however it is derived, is not necessarily a direct measure of information capacity of an image. However, it is suggested that the variability revealed in these simple tests does form the basis for a more precise evaluation of the efficiency of terrain representation methods.

REFERENCES

Bjorke, J.T. (1996) Framework for entropy-based map evaluation, Cartography and GIS 23 (2) 78-95

Feng, J. and Wang, Y. (2009) ‘No representation without information flow’, WSEAS Transations on Computers, 3 (8), 494-505

Li, Z. and Huang, P. (2002) Quantitative measures for spatial information of maps, International Journal of GIS, 16 (7), 699-709

Mendicino, G. and Sole, A. (1997) The information content theory for the estimation of the topographic index distribution used in TOPMODEL, Hydrological Processes, 11, 1099-1114

Wang, H., Wang, T., Su, S. and Li, Y. (2010) National topographic database information measuring, ISPRS Archives XXXVIII (part 4-8-2-W9), 165-170

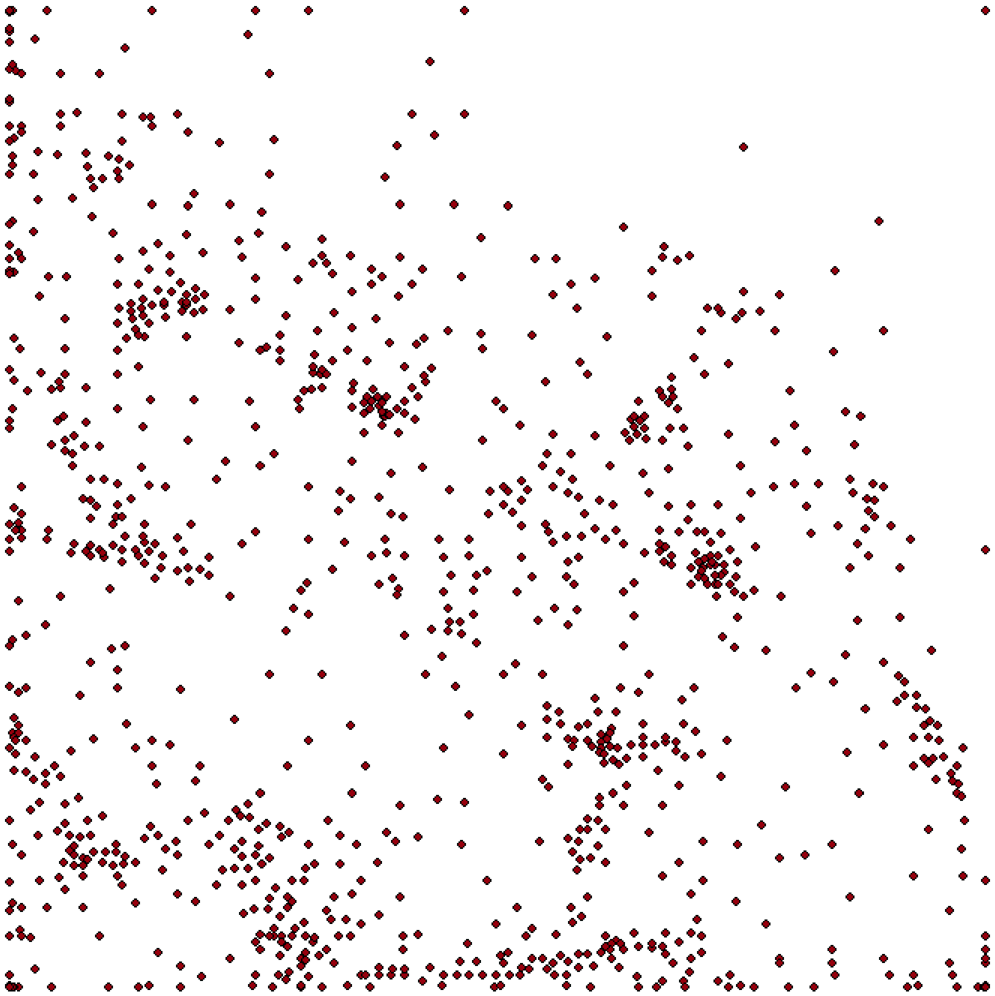

Figure 1a Spot heights across a terrain surface

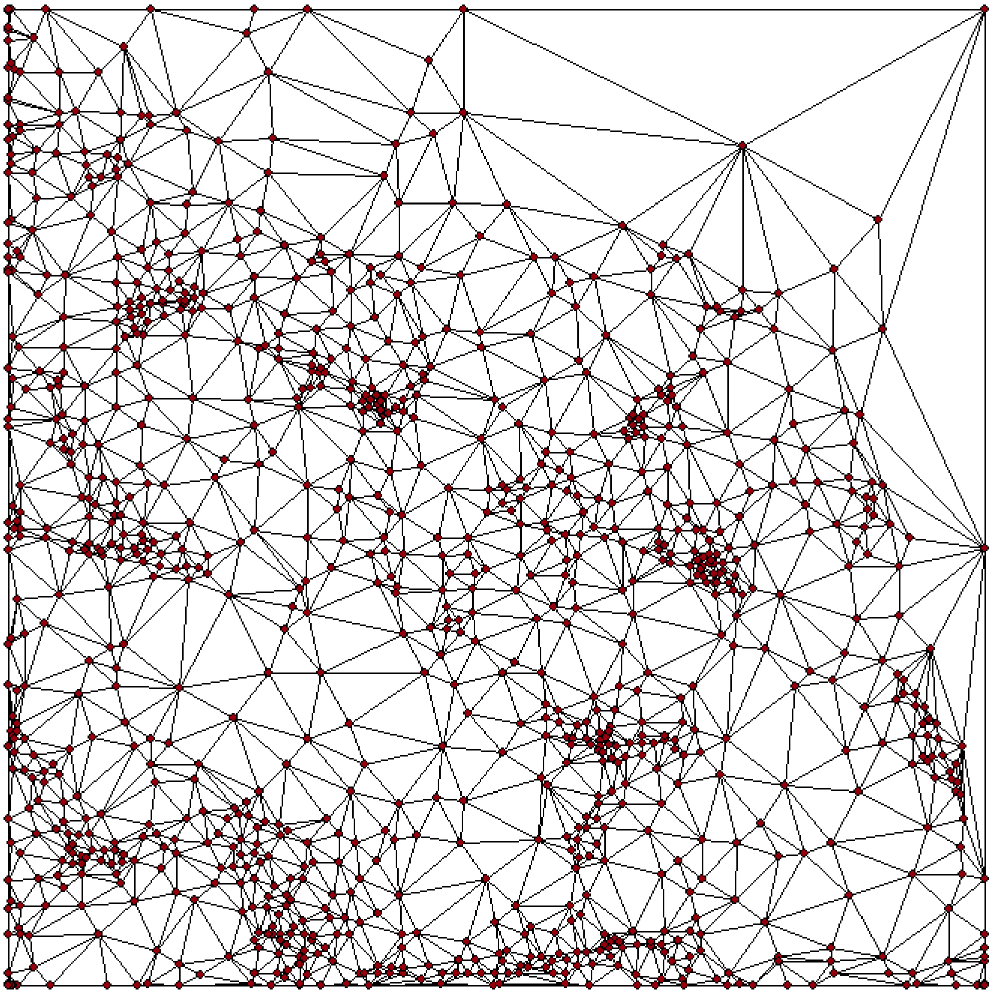

Figure 1b TIN based on the spot heights shown in Figure 1a

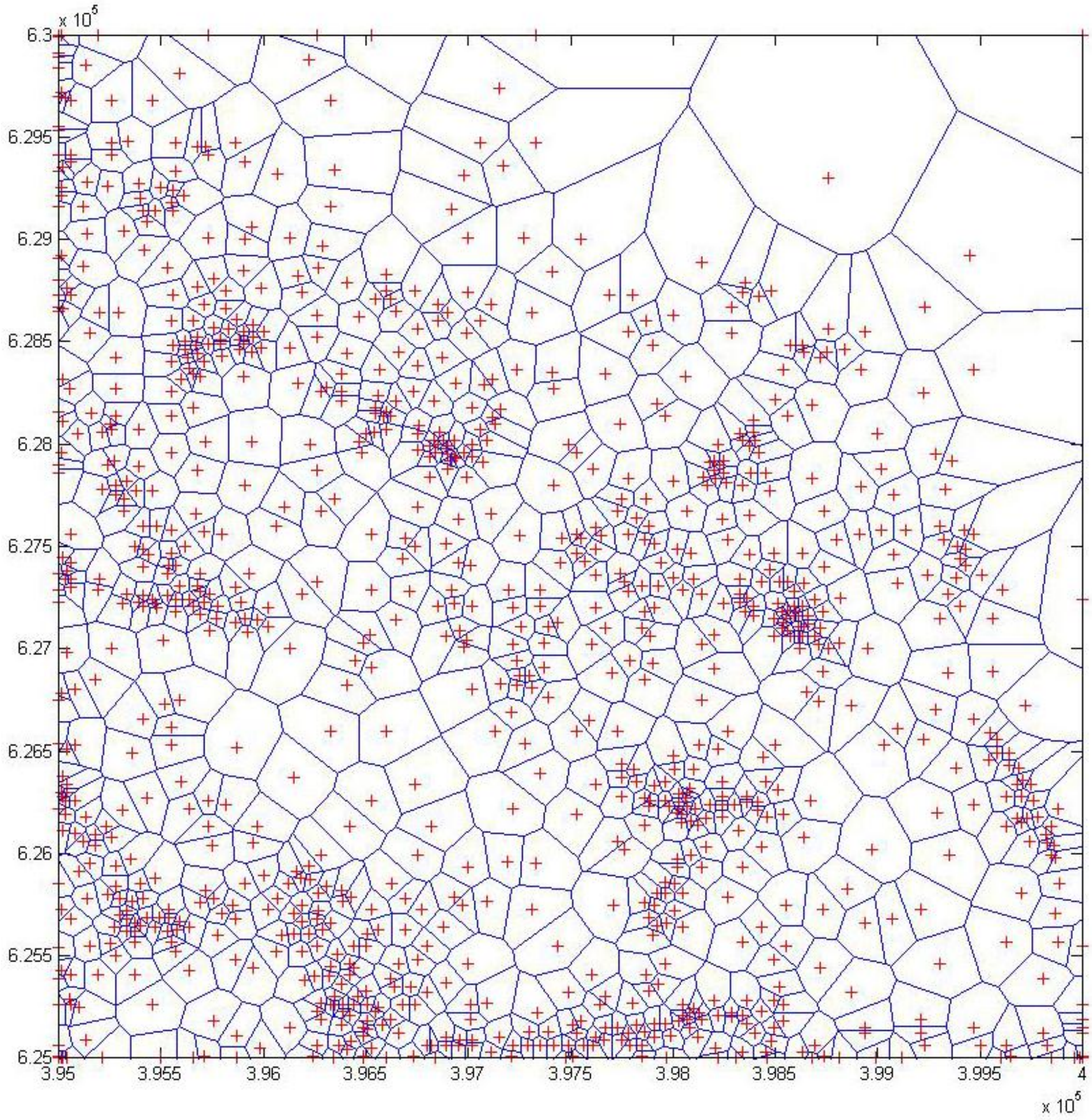

Figure 1c Voronoi diagram based on Figure 1a

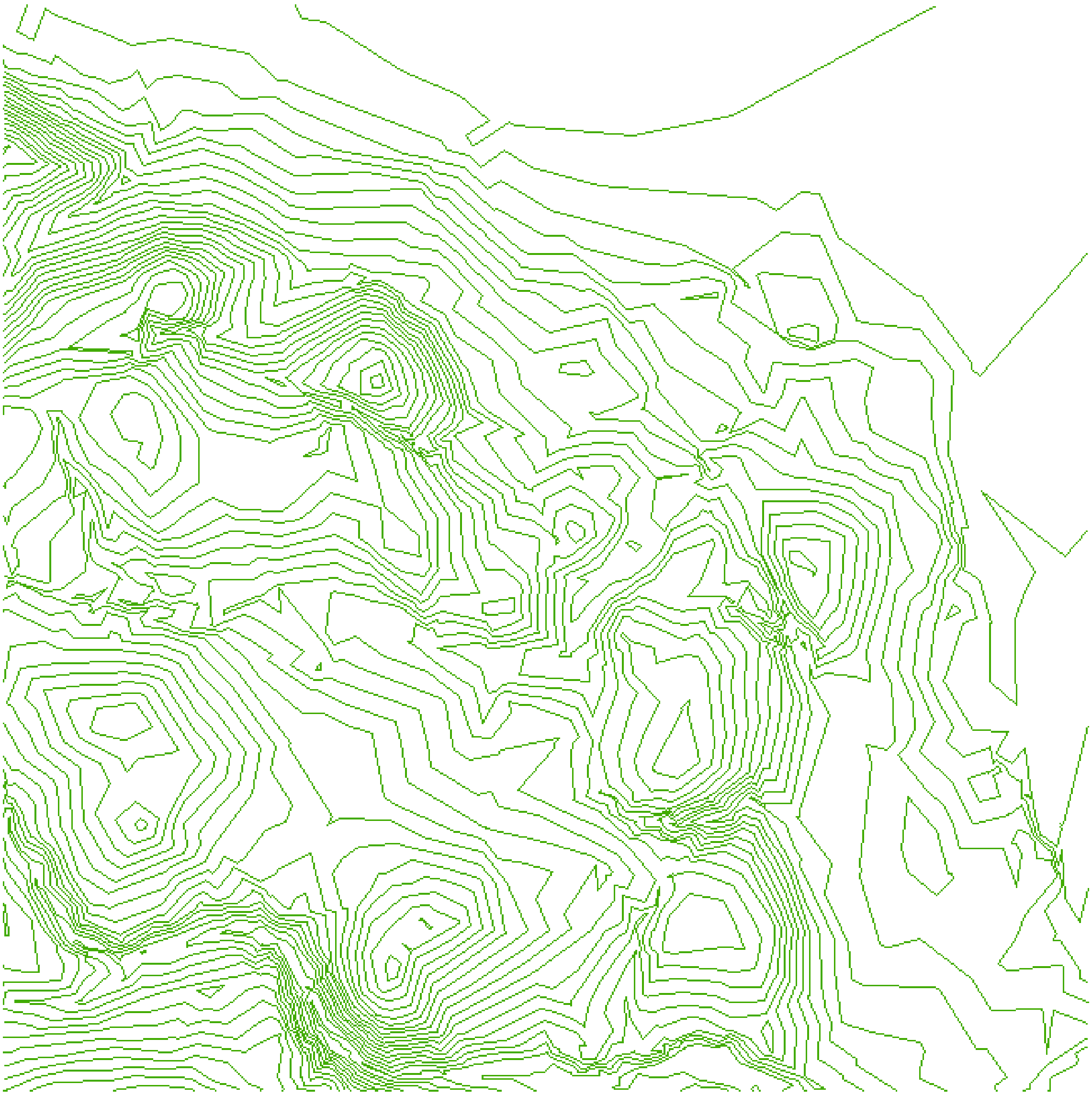

Figure 1d Contours interpolated from the TIN

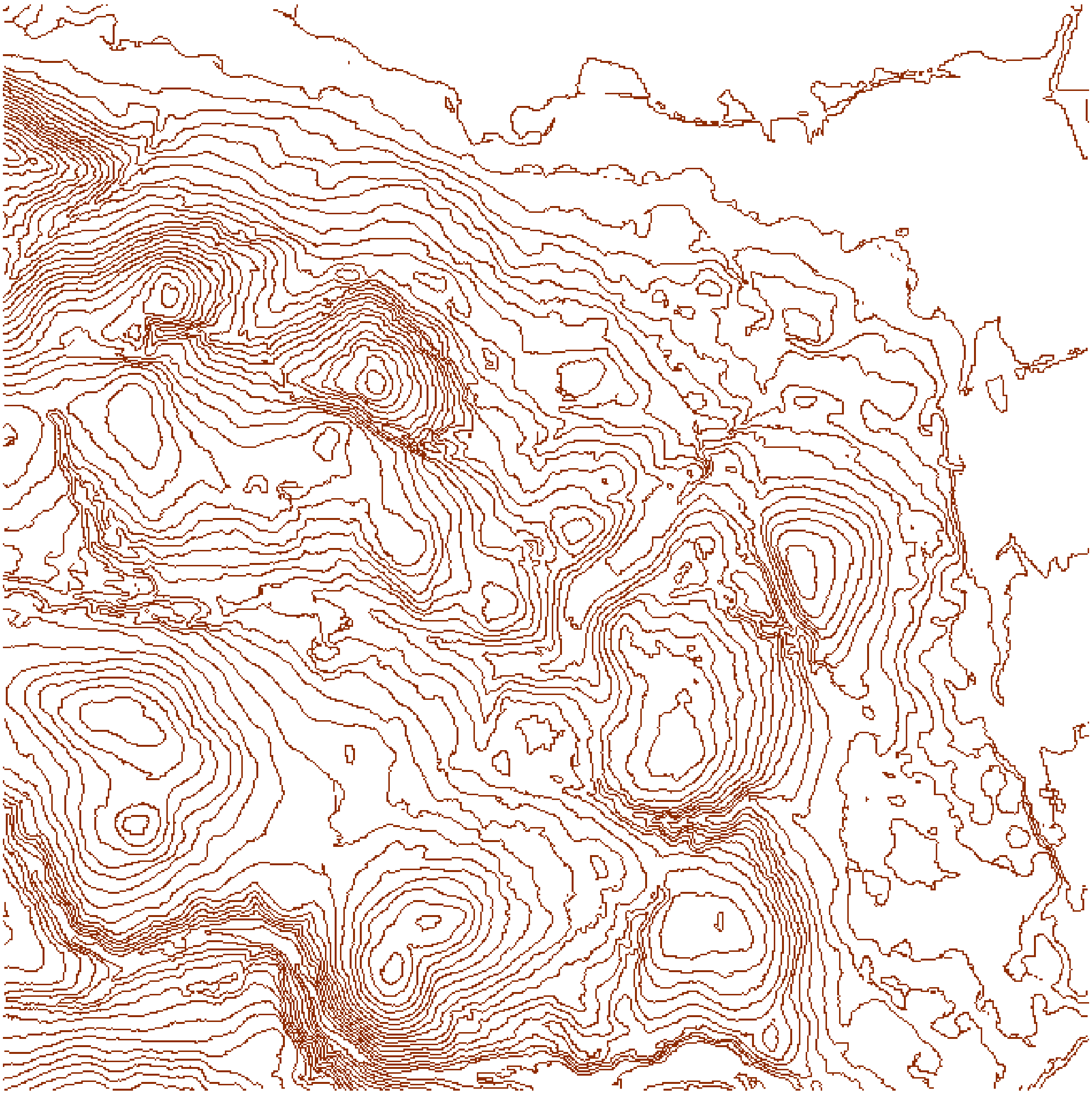

Figure 2a Contour interval 10m

Figure 2b Contour interval 50m

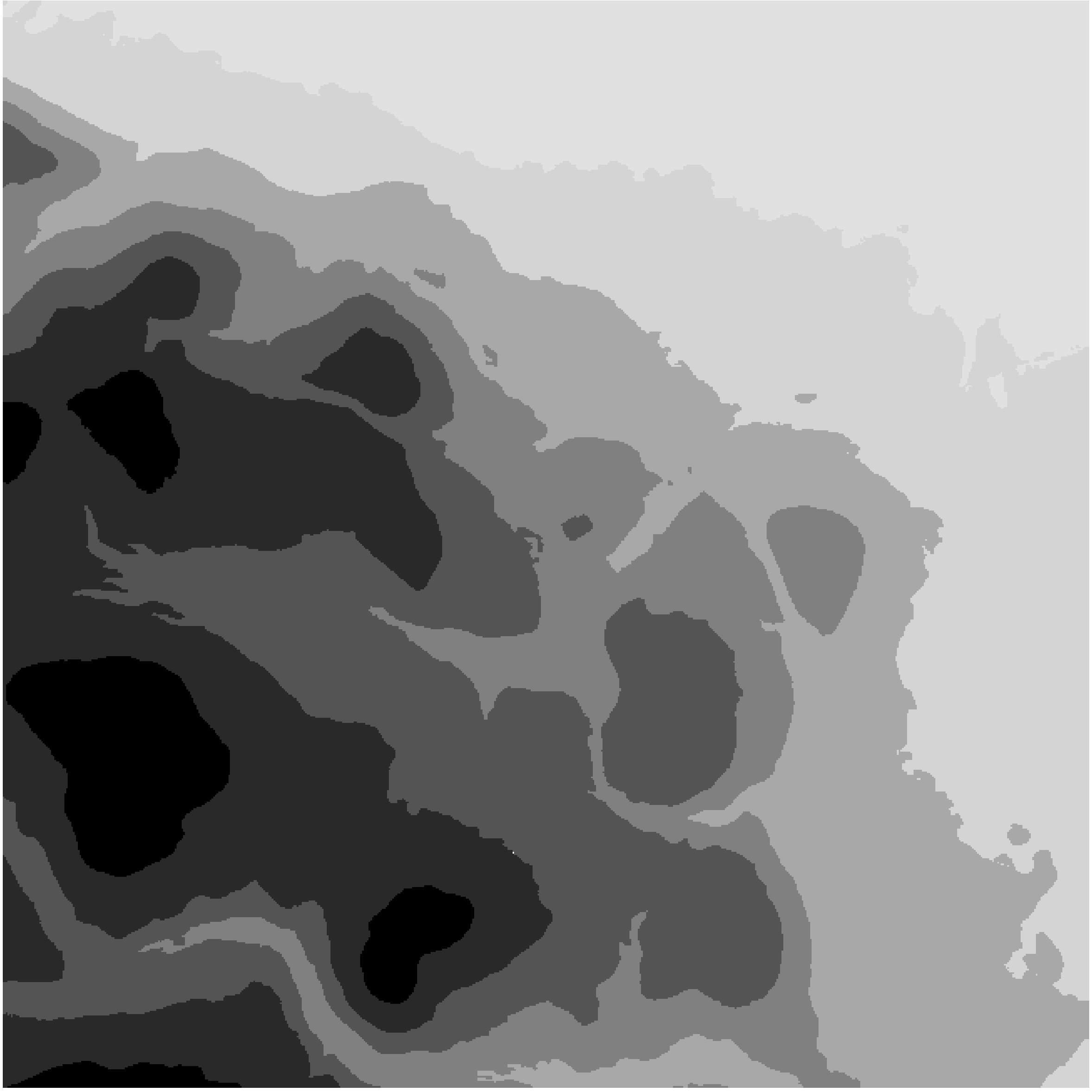

Figure 2c Hypsometric tints

Figure 2d Hillshaded surface