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A TRIANGULATED SPATIAL MODEL FOR DETECTION OF SPATIAL CHARACTERISTICS OF GIS DATA

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Abstract

The detection of spatial characteristics of GIS data is one of the key technologies in GIS applications. It plays an important role in such applications as spatial planning, spatial decision, map generalization and spatial cognition. The traditional spatial characteristics detection models are only suitable for one geometry type (point, polyline or polygon) and for one type of spatial characteristics (clustering, shape or extent, etc.). This paper presents a Delaunay triangulated irregular network based model, which adapts to all three geometry types (point, polyline and polygon) and can detect many types of spatial characteristics (clustering, bend, bottleneck, grouping and so on). Based on this model, we provide different approaches to detect different spatial structural characteristics, such as distribution extent, density and skeleton for point cluster; bend structure for line object; bottleneck area and subgroup for polygon and poly-polygon. All these methods are tested and verified by related experiments; the results are promising and satisfy the basic principle of spatial cognition. The semantic and topological properties are not considered in this model, which will be enriched in future work.

Keywords: GIS; Spatial Data Mining; Spatial Cognition; Delaunay Triangulation; Voronoi Diagram

1. Introduction

With the increasing amount of digital data stored, the demand raises to interpret and derive useful information from these ‘data mountains’. The process of extracting useful information relates to the technology of data mining, which can be defined as the discovery of interesting, implicit, and previously unknown knowledge from large databases (Frawley et al., 1991). In the last years the scope of data mining from relational databases has been extended to spatial databases. To find out the geographical characteristics and spatial structures from spatial data we need the support of spatial data mining model. Basic models of data mining are clustering, regression models, classification, summarization, link and sequence analysis (Anders & Sester, 2000). All these models are only suitable for one geometry type (point, polyline or polygon) and for one type of spatial characteristics (clustering, shape or extent, etc.). In this study, we will present a Delaunay triangulated irregular network based model, which adapts to all three geometry types (point, polyline and polygon) and can detect many types of spatial characteristics (clustering, bend, bottleneck, grouping and so on).

The main indicator for detection of spatial characteristics is distance. By analogizing to the “first law of geography”, which states that closer things tend to be more similar, Daniel et al (2003) propose a “first law of cognitive geography,” which states that people believe closer things to be more similar than distant things. Geometry in geographic space is not just Euclidean, and in fact, it is not just metric. In other words, similarity can be suggested in terms of several types of “distance,” especially when distance is understood broadly to include a variety of expressions of separation (temporal, topological, semantic, etc.). In this paper, we consider the concept of distance as metric. Under this condition, the key of detecting spatial characteristics is to develop a distance-sensitive model. Studies have shown that Delaunay TIN has some good features, such as the circumcircle of each triangle does not contain any other nodes, which making it possible to avoid the appearance of sharp angles. In addition, the outer boundary of the triangles forms a minimum convex hull, and the dual of Delaunay TIN forms another important model structure Voronoi diagram. For detection of spatial characteristics, these features can be used to search neighboring objects and detect spatial conflicts (Ware & Jones, 1997; Peng, 1995; Bader & Weibel, 1997). All of these features can be used as the mathematic basis of detection of spatial characteristics from geometry data.

The rest of paper is organized as follows. Section 2 investigates the questions of formal description of Delaunay TIN model and how to represent the general geometry objects. Section 3 is the emphasis of this paper, in this section the application of Delaunay TIN model in the detection of spatial characteristics will be discussed, such as the distribution of point cluster, the bend structure of line object, the bottleneck area of polygon and the clustering of polygon groups. Section 4 provides the conclusion with the future works.

2. Delaunay TIN Model

2.1 Formal Description of Delaunay TIN

Vector GIS data includes three kinds of geometric type: point, polyline and polygon. Delaunay Triangulation will be constructed based on these geometric features. To construct the Delaunay Triangulation, we need to identify the space domain of research area firstly, so we will insert three points Q_1, Q_2 and Q_3 into the research area. The triangle $\triangle Q_1Q_2Q_3$ formed by the inserting points should cover all the spatial objects in current research area. We can get a planar point set, which composing of Q_1, Q_2, Q_3 and the boundary points of objects within $\triangle Q_1Q_2Q_3$. Based on the point set, we construct the constrained Delaunay triangulation W. The constrained Delaunay triangulation can be denoted as triple W<V, E, T>, in which V is a non-empty point set $V=\{v_1,v_2,\ldots,v_m\}$, and E is a non-empty edge set $E=\{e_1,e_2,\ldots,e_n\}$, and T is a non-empty triangle set $T=\{t_1,t_2,\ldots,t_n\}$. The algorithm of constructing Delaunay triangulation is a hot research in computational geometry and gain lots of achievements. In this research our emphasis is on the application of Delaunay triangulation for detection of spatial characteristics, so we will employ the mature methods, such as this one developed by Peucker et al (1978).

From the view point of graph theory, the triangle network W is also a planar graph G<V, E>, in which each $e_i(e_i\in E)$ is composed of a couple of points from V and can be denoted as $e_i(v_{i0},v_{i1})$, and similarly each t_i is composed of three edges from E and can be denoted as $t_i(e_{i0},e_{i1},e_{i2})$. For any t_i, $t_j(i\neq j)$, we have $t_i\cap t_j=\varnothing$ or $DIM(t_i\cap t_j)=1$. That is to say any two triangles will not overlap each other and they can only intersecting or sharing with an edge. Fig.1 is an example of Delaunay TIN representation of point, line and polygon objects. The left one is the original representation of point, line and polygon object in vector format, and the right one is the representation of the same data with the Delaunay TIN model, in which each point object is represented as a vertex, each line object is represented as a series of edges and each polygon object is represented as a series of triangles.

Based on the Delaunay TIN model, some operations can be set up. If two triangles t_i and t_j sharing the only edge, in other words $\exists u,v:e_{iu}=e_{jv}$, we think t_i and t_j adjacent to each other. Now we can define some functions as follows:

$Neighbor(t_i,e_{ij}):T\times E\rightarrow T$, return the triangle t_j which is adjacent to the triangle t_i and they share the common edge with e_{ij};

$Neighbor(t_i):T\rightarrow T$, return a triangle set including all triangles adjacent to t_i, it can be expressed as $Neighbor(t_i)=\{ Neighbor(t_i,e_{i1}), Neighbor(t_i,e_{i2}), Neighbor(t_i,e_{i3})\}$;

$Joint(v_i):V\rightarrow T$, return the triangles which including the vertices v_i;

Condition(c):{logical expression c}→T, return the triangles satisfying the conditions c.

$Access(t_i,t_j):T\times T\rightarrow\{True,False\}$, if there being a path between any vertex of t_i and t_j in the graph G<V, E> return True; Otherwise return False.

$Begin(e):E\rightarrow V$, return the starting point v_b of edge e.

$End(e):E\rightarrow V$, return the terminating point v_e of edge e.

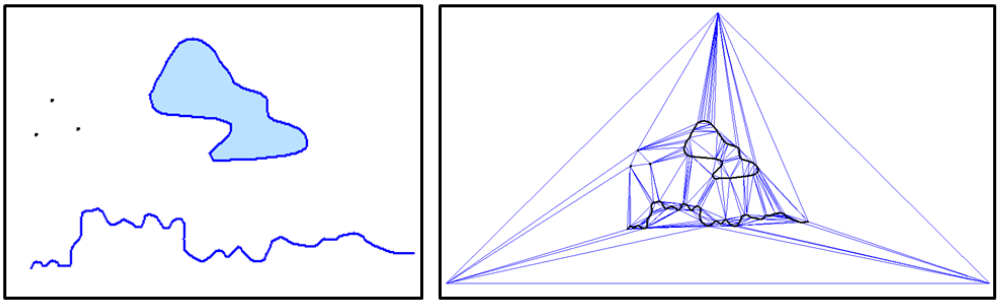


Fig. 1. Representation of geometry objects with Delaunay TIN model.

2.2 Expression of Geographical Object with Delaunay TIN Model

All features in the research area $\triangle Q_1Q_2Q_3$ can be denoted as $F_\triangle$, all points as $P_\triangle$, all lines as $L_\triangle$ and all polygons as $A_\triangle$. Then their relationships can be denoted as $F_\triangle= P_\triangle\cup L_\triangle\cup A_\triangle$. In this paper the formal model of spatial object based on the Delaunay TIN model is defined as follows.

Point object: point feature $P\in P_\triangle$ is defined as one vertex v_i of V, if P belongs to W<V,E,T>, then $P\in V$;

Line object: line feature $L\in L_\triangle$ is defined as series of edge $\{e_0,e_1,\ldots,e_n\}$, in which $e_i\in E(0\leq i\leq n)$, and $End(e_i)=Begin(e_{i+1})$ ($0\leq i\leq n-1$);

Area object: polygon feature $A\in A_\triangle$ is defined as a subset of $T:\{t_0,t_1,\ldots,t_m\}$, and for any i, $j\in[0,m]$, $Access(t_i,t_j)=True$;

Based on the Delaunay TIN model, the point, line and polygon features are represented as Fig.1 showing. Obviously, it is different with the traditional raster data model. The raster data model represents all features with only one basic element, which is grid. But the Delaunay TIN model represents features with three types of basic elements, including vertex, edge and triangle. Within the Delaunay TIN model, the triangle element plays two roles, the component of polygon feature and the bridge between neighbor objects. The triangles that playing the bridge role are distributed based on the transparent view of neighbor objects and the principle of "the nearest connection" of Delaunay TIN. That is to say the neighborhood relationship will be represented by only one triangle no matter how far between these objects. This is a good characteristic for spatial neighborhood analysis.

3. Detection of spatial characteristics

In GIS the geographical entity is usually represented as point, polyline, polygon, multipoint, poly-polyline and poly-polygon. In this section we will present some methodologies to discover the spatial structure hidden in these geometry shape based on the Delaunay TIN model. We know that the spatial characteristic is on the geographical level not on the geometric level. So the main task of detection of spatial characteristics is to discover these spatial structures hidden in geometric representation.

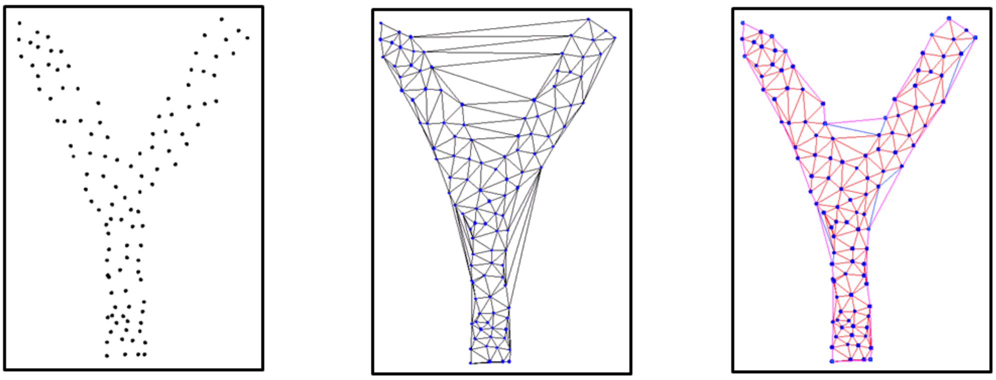
3.1 Point object

In GIS point feature has only coordination and some attributes information. Its shape characteristic can be omitted. So point feature has no spatial characteristics to be discover and we focus will be put on the multipoint feature. In the field of computational geometry, the distribution of point cluster belongs to the research of spatial neighborhood. In this section we will employ the Delaunay TIN model to extract the distribution extent, density and skeleton line characteristics of point cluster. The following discussions follow the precondition: all the points with the same semantics and belong to the same feature class.

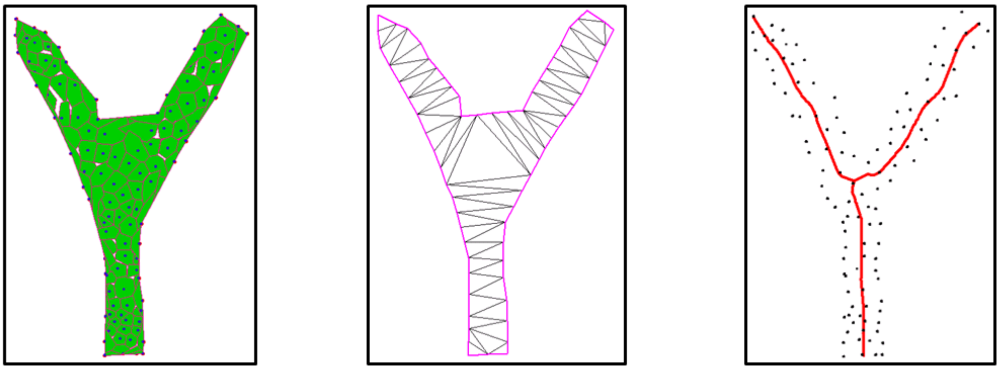
Distribution extent of point cluster. The generation of a extent from a set of points has been widely investigated (Moreira & Santos, 2007; Alani et al., 2001; Ray et al., 1997; Duckham et al., 2008; Galton & Duckham, 2006; Garai & Chaudhuri,1999). Maximillian and Antony (2009) present a systematic classification of the footprints, the algorithms used, and the types of applications they can be used for. In fact, the distribution extent of point cluster is an uncertainty problem. One basic principle is that the extent should include all the points and meet people's habits of visual perception of space, saying gestalt principle. Convex hull is usually employed to substitute the extent of point cluster (Moreira & Santos, 2007). This method may cover some concave fields containing no point. Actually the extent is constructed based on the linkage of neighbor point, and link edge should shorter than the distance of human's visual cognitive according to gestalt principle. Based on this idea, we construct Delaunay TIN on the point cluster first, and extract these edges located on the boundary as the initial extent. Now we define a distance d as the threshold of visual neighborhood. If the edge of initial extent is longer than d, it should be substituted by the other two edges which belong to the same triangle. To repeat this process until all edges shorter than d we get the extent satisfying the visual neighborhood of d. In Fig.2, the graph a the original point cluster, the graph b is the Delaunay TIN model related to this point set, and graph c is the distribution extent of this point cluster. Comparing the graph b and graph c we can discover that some long edges being deleted and the boundary of c is more close the true distribution. Actually, with different threshold d we can get different extent, but the question of which one is the best should meet the special of data condition and gestalt principle.

Distribution density of point cluster. The concept of density can be defined as the number of point in one unit (such as a grid). Conversely it can also be defined as the influence field of one point. In this study we adopt the latter definition, which is just coinciding with the characteristics of spatial competition of Voronoi diagram. We know that the Voronoi diagram is the dual of Delaunay TIN. So the calculation of distribution density can be finished by two steps. One is to construct Delaunay TIN based on the extent got by the former paragraph. The other is to construct Voronoi diagram and calculate the area ai of each cell within the Voronoi diagram. Then the density of each point can be defined as the reciprocal of its related cell's area 1⁄ai. The small value of 1⁄ai means the point pi occupying a large space, then the density is small. On the contrary, the greater the value 1⁄ai, the greater the density. Fig.2 (d) shows the distribution density of the point cluster, from the figure we can get some institute impression that the greater the cell the smaller the density.

Distribution skeleton line of point cluster. From the discussion above we know that the extent can describe the distribution characteristics of point cluster in 2-demention space. In some case, we want even more simple description of a point cluster, such as the skeleton line in 1-demention. Similarly, we construct Delaunay TIN within the extent, in this step only the points on the boundary of extent participate the constructing, and others within the extent should be excluded. The result Delaunay TIN is as Fig.2 (e) showing. Now based on the extent polygon we can extract its skeleton line, which is just the distribution skeleton line of point cluster. The algorithm of extracting skeleton based on Delaunay TIN is developed by Ai and Oosterom (2002). Fig.2 (f) shows the result of skeleton line extracting based on Delaunay TIN model. From which we can see that the point cluster is distributed like a letter “Y”.



a. a point cluster b. TIN model c. get boundary by cutting long edges



d. compute density e. make TIN on boundary points f. get skeleton line

Fig. 2. The detection of spatial characteristics of point cluster (range, density and center)

3.2 Line Object

For line object, the structural information of geographical characteristics is hidden in its bends. So the detection of bend is a hot topic in GIS (Plazan et al., 1995; Wang & Muller, 1998). In this study we provide an approach based on Delaunay TIN to detect the bend characteristics of geometry line.

Firstly, we insert three points Q_1, Q_2 and Q_3 to define the data extent and make Delaunay TIN, all the points of line and Q_1, Q_2, Q_3 participate this process. To facilitate the analysis, we categorize the triangles into four types.

Type Ⅰ: The triangles which connect the vertex of Q_1 or Q_2 or Q_3;

Type Ⅱ: The triangle which has only one neighbor (that is Neighbor(t_i)=1), and not connect to Q_1, Q_2 and Q_3;

Type Ⅲ: The triangle which has two neighbor (that is Neighbor(t_i)=2), and not connect to Q_1, Q_2 and Q_3;

Type Ⅳ: The triangle which has three neighbor (that is Neighbor(t_i)=3), and not connect to Q_1, Q_2 and Q_3.

Different type of triangles will play different roles in the detection of bend. The first type of triangles (type Ⅰ) are external triangles, of which the edge that locating on the curve serves as the entrance of bend. Type Ⅱ、Ⅲ and Ⅳ are valid triangles which will cover the whole area of one bend. The second type (type Ⅱ) serve as the bottom of one bend, and the third type (type Ⅲ) serve as the component of one bend, and the forth type (type Ⅳ) serve as the bifurcation of two sub-bends.

Based on the triangle’s different classification and function, we can search one curve’s hierarchical bends series. We enter the bend field from type Ⅰ triangle, and then we delete the current type Ⅰ triangle and get its neighbor triangle as current triangle. If current triangle is type Ⅲ, the search path is unique; if current triangle is type Ⅳ, we get a branch of sub-bend; if current triangle is type Ⅱ, we arrive at the end of the bend. The process of search can be recorded by a binary tree structure (Ai et al., 2000). Additionally, some measure elements can be define based on the bend structure, such as bend depth (the distance from the bottom to the entrance line), bend width (the width of the entrance line), and the area and so on. Fig.3 is an example of detection of bend characteristics of line object. The left one is the Delaunay TIN representation of the line object, and the right one is the result of bent detection, in which the green and blue parts are little bends extracted with a certain threshold.

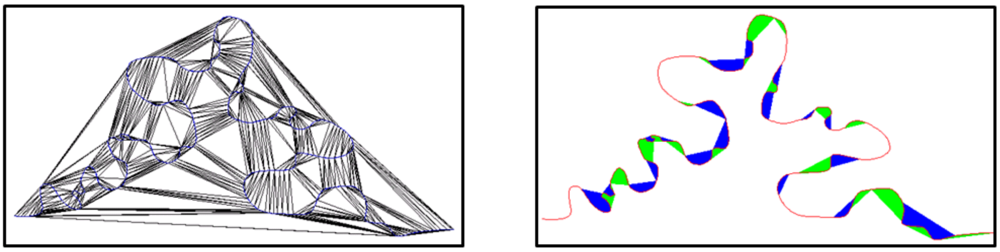


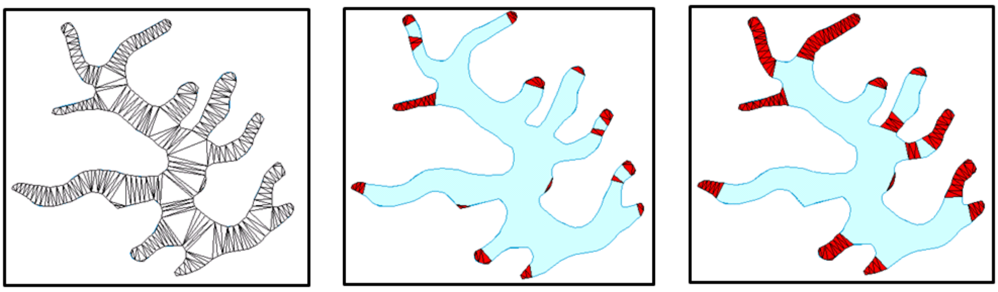
Fig. 3. The detection of bend characteristics of line object.

For line cluster, its structural information can be differentia into two types (Cecconi, 2002): the humanities structure such as the road connectivity within one road network, and the natural structure such as the river hierarchy within river network. For artificial line network, its structure can be extract from its semantic characteristics, such as highway, state way, province way and rural road. For natural network the extraction of its structural geographical characteristics is complex. Such as for river network the hierarchical analysis is a sophisticated and hot topic in hydrology and GIS area. Ai et al (2007) provide a Delaunay TIN based method to get the structural hydrology information, and the process is not discussed here in detail.

3.3 Polygon Object

For single polygon object, its structural information of geographical characteristics is hidden in its bends or within its shape (such as bottleneck area). The method of bend detection of polygon object is similar to this of line object. So we will not detail it again in this section. Also the bottleneck area is important structural information for area object, which determines the form of a river on the map. If the river is wide enough it will be represented as a polygon object, otherwise it will be represented as a line object. How do we know which one will be represented as polygon or line? This question depends on the detection of area object’s bottleneck field. Now we put the focus on the detection of bottleneck area of single polygon object.

Firstly we construct Delaunay TIN based on the polygon object. Obviously the bottleneck area is within the polygon. So we extract the internal triangles within the polygon as the research triangles. This can be executed by the operation of Condition(c). Fig.4 (a) shows the result of the condition operation, of which the triangles within polygon were extracted. The so-called bottleneck area is this narrow area. We objective is find out which area is narrow, such as these areas which width are small than a threshold d. Secondly we calculate every triangle’s height. All triangles within the polygon have the common feature that there are two vertexes of triangle being continuous on the boundary of polygon, and the other vertex being far from these two continuous vertexes (at least in one point interval). Then we can make vertical line from the isolated vertex to its opposite edge. To get the bottleneck area we just need to extract these triangles whose’ height small than d. Fig.4 (b) and (c) shows the result of bottleneck detection with different thresholds. Fig.4 (b) with a small threshold and the bottleneck area is small; Fig.4 (c) with a great threshold and the bottleneck area is great.



a. make TIN b. get bottleneck with d_1 c. get bottleneck with $d_2(d_2 > d_1)$

Fig. 4. The detection of bottleneck characteristics of polygon object.

Now we will discuss the spatial characteristics of polygon group. Important structural information of polygon group is clustering (Regnauld, 1997). For GIS data the clustering operation is usually for the same feature category. In this study we provide a methodology based on Delaunay TIN model to realize the polygons’ clustering. For a set of objects, from a similar category, we construct the constrained Delaunay triangulation. The triangles connecting different polygons are considered which implies that those triangles within and outside the polygons but with three vertices located in the same polygon will be removed from the triangulation. For the remaining triangles we calculate every one’s height. The method is the same as the former paragraph. Define a neighborhood tolerance distance d, and then select those triangles whose local distance between the connected object polygons is shorter than d. and then the polygons related to the selected triangles can be grouped into one cluster, see Fig.5. The left one is the Delaunay TIN for polygon groups, and the right one is the result of clustering. From which we can see that all the polygons are classified into seven groups based on visual distance, the result satisfy the basic principle of spatial cognition.

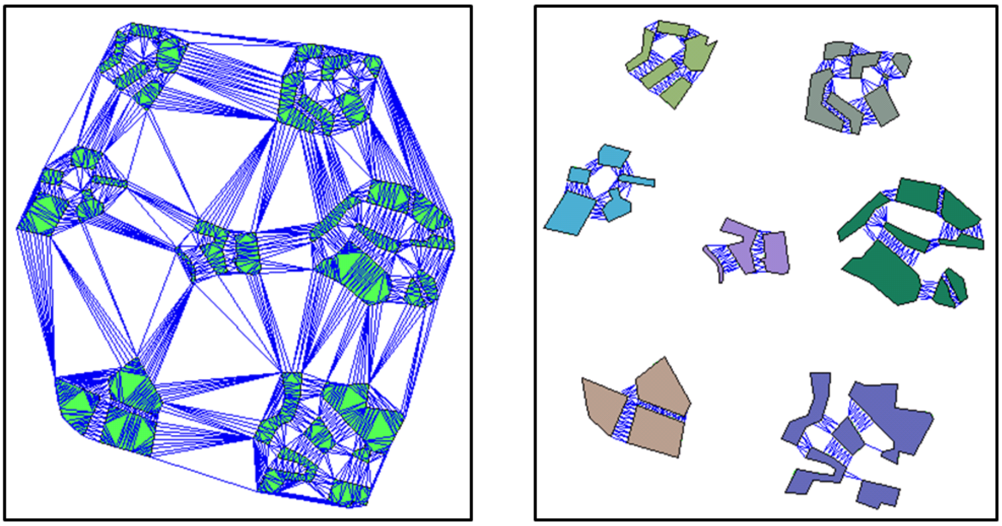


Fig. 5. The detection of grouping characteristics of polygon cluster.

4. Conclusion and Future Work

This paper presented a triangulated spatial model to automatically discover spatial characteristics of in spatial databases. In this study we concentrate on geometric properties but not refer to semantic properties. Under the condition of “first law of cognitive geography” presented by Daniel et al (2003), Delaunay TIN is a powerful tool for spatial adjacency analysis. In order to meet the needs of spatial data analysis, the Delaunay TIN model was described as a triple W<V, E, T>, in which V is a non-empty point set $V=\{v_1,v_2,\ldots,v_m\}$, and E is a non-empty edge set $E=\{e_1,e_2,\ldots,e_n\}$, and T is a non-empty triangle set $T=\{t_1,t_2,\ldots,t_n\}$. The geometry feature point, polyline and polygon were denoted as a vertex, a set of edges and a set of triangles. Then the spatial relationship of objects can be defined based on the unified formal TIN model, for example, the neighbor () operator was designed to get the neighborhood relationship between triangles and the access () operator was designed to get the accessibility relationship between triangles, and other operations and so on.

Based on single or combination of these operations, some typical spatial characteristics can be detected, such as cluster, bend, bottleneck and so on. For point cluster, their distribution extent, distribution density and distribution skeleton were discovered. By cutting the long edges of related triangles the truly extension was approximated, even the “Y” liked distribution the concave shape can be well maintained. By the dual graph of TIN, the Voronoi structure was utilized to extract the distribution density of point cluster. By the skeleton operation the distribution center of point cluster was extracted. For line object, one import spatial characteristic is bend. In this paper we distinguish the triangles in TIN into four types, and based on different type of triangle the bend characteristics were accurately identified. Compared with the infection point based method and others, the result can better meet the human spatial cognition. For polygon object, based on visual adjacency distance the bottleneck area was extracted, which is import for spatial decision such as the determination the point of traffic congestion, and is significant for map generalization such as determination the area of skeleton. For polygon group, the polygons were clustered into different sub-groups through distance threshold. From all these experiments (Fig.2 to Fig.5), we can see that the results can satisfy the basic principle of spatial cognition and this approach proposed in this paper is sensible and promising.

Further investigations on the procedure to consider the semantic properties have to be done. For example, if the semantic property was taken into account during the polygon clustering, the whole polygon area can be divided into different functional area, such as industrial areas, commercial areas, residential areas and so on. In addition, the detection process has to be extended to different spatial characteristics (temporal, topological, etc.). Methods for data mining of different spatial characteristics have to be developed.

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