SMOOTHING OR PLANING: APPLICATION OF THE MEADIAN AUTO-ADAPTATIVE SPATIAL FILTER ON SATELLITE IMAGES

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1. USING THE MEADIAN: WHY?
The MeAdian comes from previous works in Geography and Statistics in 2001. The initial aim was to obtain a local spatial filtering method which enables to process a robust center based on a combination of Mean for smoothing and Median for planing (Josselin & Ladiray, 2002, 2004). The method should be automatic. Another reason is that, when processing local analysis, we often have to deal with only few individuals. So the robustness can be necessary to compare results. These investigations lead us to find the early works of Laplace (1818) who already thought about a linear combination of both Lp norms. He concluded it was not interesting to go further in such research. This contributed to separate these two complementary ways to find a center, that defined a large part of the statistics history. In this paper, we improve the MeAdian filtering for contour detection.

2. HOW TO BUILD THE MEADIAN?
Mathematically, we use a resampling method (non parametric bootstrap, Efron & Tibshirani, 1993) to estimate the robustness of the Mean and the Meadian. The principle is rather simple: the inverse of the respective robustness is used to weight the Mean and Median values. Laplace proposed to add a covariance factor (fig. 1). A center remained stable among all the boostraped distributions has a low variance and a high robustness. If we accumulate all the samples, the final distribution (for instance of 100 000 individuals if we make 1000 iterations of a distribution including 100 individuals) is perfectly similar to the initial one (fig. 2).

3. MEADIAN ROBUSTNESS FACED TO OTHER CENTRAL VALUES
Beyond the variance, the relative efficiency of an estimator is defined by the ratio between its variance and the one of the best estimator tested in the same conditions. An efficiency equal to 100% means that the estimator has been the most robust. Calculated using R, the results show that, for most of the theoretical tested distributions, the MeAdian obtains quite good scores whatever distribution is considered (fig. 3). Moreover, the Mean and the Median have very bad efficiencies for Cauchy resp. Gaussian distributions. This suggest the use of the adaptative MeAdian: when the Mean is more robust, the MeAdian tends to it, to the Median otherwise.

4. RESULTS: MEADIAN FILTERING ON SATELLITE IMAGES
For the first time, the MeAdian filter is used on images, according to several parameters:
the size of the local window (from 3x3 to 7x7 cells),
various iterations on filtering,
change of the weight of variances in the linear combination.
The results show the double effect of the MeAdian: a combination of smoothing and planing according to the local distributions encountered. Even with various pixel values, homogeneous areas are smoothed, whereas outliers are eliminated from other local trends (figure 4). The difference of the Mean and MeAdian filters emphasizes the boundaries between homogeneous and heterogeneous areas. So this kind of filter can be used for texture analysis and possibly for contouring (figure 5).
Laplace MeAdian = (1-C)\mu - C\times M

\text{with } \mu: \text{the arithmetic Mean,} \\
M: \text{the Median,} \\
V(\mu): \text{the Mean Variance,} \\
V(M): \text{the Median Variance.}

\text{and } C = \frac{\text{V(\mu) - Cov}}{\text{V(\mu) + V(M) - 2*Cov}}

\text{and } \text{Cov} = \text{cov(M, \mu)}

Josselin-Ladiray MeAdian = (1-C)\mu - C\times M

\text{and } C = \frac{\text{V(\mu)}}{\text{V(\mu) + V(M)}}

\text{Figure 1. The MeAdian mathematical formalism.}

\text{Figure 2. Comparison different centrality values robustness according to different statistical distributions.}

<table>
<thead>
<tr>
<th>Random dist.</th>
<th>Mean</th>
<th>Median</th>
<th>JL_MeAdian</th>
<th>L_MeAdian</th>
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<td>17.06</td>
</tr>
</tbody>
</table>

\text{Figure 3. Initial observed distribution and distribution of all the individuals of the bootstraped distributions (N=1000).}
Figure 4. A MeAdian filter 3x3.
Figure 5. Difference between the Mean and the Median filters (3x3).