CONVERSION OF COORDINATES BETWEEN THE GAUSS-KRÜGER AND UTM CARTOGRAPHIC PROJECTION SYSTEMS ON TOPOGRAPHIC MAPS IN ROMANIA AND REPUBLIC OF MOLDAVIA FOR GIS USE

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OBJECTIVES
The conversion of coordinates between different systems of cartographic projections is necessary in order to achieve a unitary system database of a Geographic Information System mapping. UTM (Universal Transverse Mercator) projection is a variant of the Gauss-Krüger projection used in the United States and other countries. By means of automated mapping, a map made based on Gauss-Krüger projection system (official in Romania) is converted into the UTM projection system (official in the Republic of Moldavia). It is necessary to consider how transfer of the map content from Gauss-Krüger projection (reference ellipsoid Krasovski – 1940) into UTM projection (reference ellipsoid WGS - 84) is performed by using coordinates transformation algorithms and georeferencing processes in the graphical environment of a GIS program.

For example, consider the topographic map sheet at a scale of 1:100 000 labeled L-35-33 (Sălăjeni), drawn in Gauss-Krüger projection. A number of characteristic points were selected and indicated by the four corners of the geodetic trapezoid and rectangular grid intersections at every 10 km; they are to be converted from Gauss-Krüger projection to UTM projection (Figure 1).

Finally, the georeferencing accuracy (± 0,2 mm) and the sheet map (L-35-33) converted from Gauss-Krüger projection in UTM projection, are evaluated; the graphical information is checked against the attached sheet map labeled L-35-45 (Husi-3) at a scale of 1:100000 which was drawn and spatially referenced in UTM projection.

Figure 1 - Characteristic points set for georeferencing the map at a scale of 1:100000

METHOD USED
• Conversion of rectangular plane Gauss-Krüger coordinates (X, Y) of the characteristic points to ellipsoidal geographic coordinates (φ, λ) on the reference ellipsoid Krasovski – 1940.
The calculation was made by means of varying-coefficient method,

\[ \varphi = \varphi_1 + \Delta \varphi; \lambda = \lambda_0 + l \]

using:

1 - the latitude of the point on the abscissa axis (OX):

\[ \varphi_1 = \frac{X}{\alpha} + \beta \sin \frac{2X}{\alpha} + \gamma \sin \frac{6X}{\alpha} + \varepsilon \sin \frac{8X}{\alpha} \]

\[ \alpha = \frac{a+b}{2} \left( 1 + \frac{1}{4} n^2 + \frac{1}{64} n^4 + \ldots \right) \]

\[ \beta = \left( \frac{3}{2} n - \frac{27}{32} n^3 + \frac{269}{512} n^5 + \ldots \right) \]

\[ \gamma = \left( \frac{21}{16} n^2 - \frac{55}{32} n^4 + \ldots \right) \]

\[ \delta = \left( \frac{151}{96} n^3 - \frac{417}{128} n^5 + \ldots \right) \]

\[ \varepsilon = \left( \frac{1097}{512} n^4 + \ldots \right) \]

\[ n = \frac{a-b}{a+b} \]

- X: the abscissa of the point considered:
- \( a, b \): the long, respectively the short semi-axis of the ellipsoid Krasovski 1940
- \( \lambda_0 \) – the axial meridian longitude, corresponding to 6° longitude zone
\[ \Delta \varphi' = \Delta \varphi'' = A_2 Y''^2 + A_4 Y''^4 + A_6 Y''^6 \]
\[ l'' = B_1 Y' + B_3 Y'^3 + B_5 Y'^5 \]

A2 = \(-10^{10} \frac{\rho^n}{2N_1^2} t_1 (1 + \eta_1^2)\)

A4 = \(10^{20} \frac{\rho^n}{24N_1^4} t_1 (5 + 3t_1^2 + 6\eta_1^2 - 6t_1^2\eta_1^2 - 3\eta_1^4 - 9t_1^2\eta_1^4)\)

A6 = \(10^{30} \frac{\rho^n}{720N_1^6} t_1 (-61 - 90t_1^2 - 45t_1^4 - 107\eta_1^2 + 162t_1^2\eta_1^2 + 45t_1^4\eta_1^2)\)

B1 = \(10^5 \frac{\rho^n}{N_1 \cos \varphi_1}\)

B3 = \(-10^{15} \frac{\rho^n}{6N_1^3 \cos \varphi_1} (1 + 2t_1^2 + \eta_1^2)\)
\[ B5 = 10^{25} \frac{\rho''}{120N_1^5 \cos \varphi_1} (5 + 28t_1^2 + 24t_1^4 + 6\eta_1^2 + 8\eta_1^2t_1^2) \]

\[ N = a / \sqrt{1 - e^2 \sin^2 \varphi} \]

\[ t = \tan \varphi \]

\[ \eta^2 = e^2 \cos^2 \varphi \]

\[ \rho'' = 206 264, 806 247 \]

\[ Y' = 10^{-5} (Y - 500 000, 000 \text{ m}) \]

The geographical coordinates (\( \varphi, \lambda \)) on the Krasovski 1940 ellipsoid of the characteristic points and calculated based on the Gauss plane rectangular coordinates (x, y) with an accuracy of \( \pm 0''0.001 \), are indicated in the Table 1.

Table 1.
Conversion of the ellipsoidal geographic coordinates (φ, λ) on the Krasovski - 1940 reference ellipsoid to ellipsoidal geographic coordinates (φ, λ) on the WGS-84 reference ellipsoid

The conversion is made by means of Molodenski simplified formulas that allow the direct passage of the geographical coordinates of the points on the two ellipsoids (φ, λ), by applying the differences between the ellipsoidal latitude and longitude:

\[ \Delta \phi = \left\{ \begin{array}{l} \Delta \phi_x \cos \gamma \cos \phi + \Delta \phi_y \cos \phi + \Delta \phi_z \sin \phi \cos \phi \end{array} \right\} \sin \phi \cos \phi \]

\[ \Delta \lambda' = -\Delta X \sin \lambda + \Delta Y \cos \lambda \]

where:

\[ \Delta X, \Delta Y, \Delta Z \]

are the translation constants of the Cartesian coordinates between the two ellipsoids;

\[ M \]

is the meridian radius of curvature.
• N – principal radius of curvature; 

\[ \Delta a = (a_l - a_S) \] – the difference between the long semiaxes of the ellipsoids; 

\[ \Delta f = (f_l - f_S) \] – the difference between the values of the flattening of the two ellipsoids.

In Romania area, the translation parameters between the two ellipsoids (Krasovski 1940 and WGS-84) are

\[ \begin{align*}
\Delta x &= 28 \text{ m} ; \\
\Delta y &= -121 \text{ m} ; \\
\Delta z &= -77 \text{ m} \\
\end{align*} \]

determined on an accuracy of ± 3 - 5 m.

The data obtained for the characteristic points are indicated in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>No. point</th>
<th>Krasovski -1940 ellipsoidal geographical coordinates</th>
<th>WGS-84 ellipsoidal geographical coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \varphi (^\circ',''') )</td>
<td>( \lambda (^\circ',''') )</td>
</tr>
<tr>
<td>1</td>
<td>47 20 00,0000</td>
<td>28 00 00,0000</td>
</tr>
<tr>
<td>2</td>
<td>47 20 00,0000</td>
<td>28 30 00,0000</td>
</tr>
<tr>
<td>3</td>
<td>47 00 00,0000</td>
<td>28 00 00,0000</td>
</tr>
<tr>
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<td>47 00 00,0000</td>
<td>28 30 00,0000</td>
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<td>6</td>
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<td>7</td>
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<td>20</td>
<td>47 00 53,2771</td>
<td>28 26 48,0795</td>
</tr>
</tbody>
</table>

• Conversion of the ellipsoidal geographical coordinates\((\varphi, \lambda)\) from the WGS-84 reference ellipsoid to UTM\((N, E)\)/WGS-84 flat rectangular coordinates:

\[
X = \beta + \frac{1}{2} N \sin \varphi \cos \varphi \frac{\rho''''^4}{\rho^{12}} + \frac{1}{24} N \sin \varphi \cos^3 \varphi \left( 5 - \frac{t^2}{4} + \frac{\eta^2}{6} + \frac{4 \eta^4}{5} \right) \frac{\rho''''^4}{\rho^{12}}
\]
The relations are written shortened as a consequence of introducing the names of the coefficients: A2, A4, A6 for the X abscissa and B1, B3, B5 for the Y ordinate:

\[
X = X_0 + \Delta X = \beta + A_2 l^2 + A_4 l^4 + A_6 l^6 \quad [\text{m}]
\]

\[
Y = B_1 l + B_3 l^3 + B_5 l^5 \quad [\text{m}]
\]

* \( \beta \) – the length of the meridian arc from Equator to \( \phi \) latitude:

\[
\beta = \gamma \omega = \lambda \phi - \delta \lambda \cos \phi + \xi \delta \phi - \gamma \phi \gamma \phi + \xi \xi \gamma \phi
\]

where P, Q, R, S, T are constant coefficients, calculated depending on the geometric parameters (a, \( e^2 \)):

\[
l = 10^{-4} (\lambda - \lambda_0)''
\]

\[
A_2 = \frac{10^8}{2 \rho''^2} N \sin \phi \cos \phi
\]

\[
A_4 = \frac{10^{16}}{24 \rho''^4} N \sin \phi \cos^3 \phi (5 - t^2 + 9 \eta^2 + 4 \eta^4)
\]

\[
A_6 = \frac{10^{24}}{720 \rho''^6} N \sin \phi \cos^5 \phi (61 - 58 t^2 + t^4 + 270 \eta^2 - 330 \eta^2)
\]
\[ B_1 = \frac{10^4}{\rho''} N \cos \varphi \]
\[ B_3 = \frac{10^{12}}{6 \rho'^{13}} N \cos^3 \varphi (1 - t^2 + \eta^2) \]
\[ B_5 = \frac{10^{20}}{120 \rho'''^5} N \cos^5 \varphi (5 - 18t^2 + t^4 + 14\eta^2 - 58t^2\eta^2 + 13\eta^4 - 64t^2\eta^4) \]

The conversion of the GAUSS (X,Y) / WGS – 84 flat rectangular coordinates to UTM (N,E) / WGS – 84 flat rectangular coordinates is a result of using the scale coefficient 
\( (k_0 = 0.9996) \)

\[ N = Xk_0 \]
\[ E = Y_0 + Yk_0 = 500000m + Yk_0 \]

**RESULTS**

The final results of the coordinate conversion are indicated in *Table 3*.

*Table 3*

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In order to evaluate the quality of the georeferentiating procedure of the map sheet, the map labeled L-35-33 (Sălăjeni) which coordinates were converted to UTM projection, is connected to the map attached to the south labeled L-35-45 (Husi-3), drawn in UTM projection at a scale of 1:100000. The rectangular grid corresponding to UTM projection every 10 km (*Figure 2*) is drawn, and then the continuity of planimetric and altimetric features, as well as of the rectangular grid across the two adjacent geodesic trapezoids was verified, within the same accuracy of the initial product.
CONCLUSIONS

• The stages of converting the Gauss–Krüger (Krasovski 1940) flat rectangular coordinates to UTM (WGS-84) flat rectangular coordinates can be summarized as follows:
\[(X,Y)_{\text{Gauss}} \rightarrow (\phi, \lambda)_{\text{KA}} \rightarrow (\phi, \lambda)_{\text{WE}} \rightarrow (N,E)_{\text{UTM}}\]

- Matching the graphic information on the adjacent map sheets can be checked by analyzing the continuity of the planimetric and altimetric features in relation to the set graphic accuracy.
- The automated process of converting maps from one system of cartographic projection to another by maintaining the graphic accuracy of the initial product allows easy handling of the engineering issues, as well as unitary approach of the data of a Geographical Informational System.