Old Maps Georeferencing – Overview and a New Method for Map Series

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Abstract. Old maps are often the basic source of modern environmental studies focused on landscape reconstruction. Georeferencing of these maps is a key activity in the implementation of these studies. It is necessary to know the theory of the basic methods of georeferencing. This paper summarizes the current knowledge of methods, and further proposes a new method for the georeferencing of multiple sheet map series. It is based on the overall adjustment of all transformation coefficients with conditions of the map edges continuity.

Keywords: old maps, georeferencing, adjustment

1. Introduction

Recently, old maps are increasingly used as a base layer for various environmental applications. With the expansion of map servers, web map services and APIs of mapping portals they are increasingly used as an essential base layer for monitoring changes in the landscape or as a source of information for the reconstruction of the original landscape. For a long time, the biggest problem with the inclusion of old maps in these applications is being solved; their georeferencing.

At the department of mapping and cartography we are working on the problematics of georeferencing of old maps for a long time. Some results of our research can be found in (Krejci 2009) or (Cajthaml 2010).

1.1. Global Transformation Methods

There are many methods of georeferencing of old maps. Core group of these transformations are global transformation methods. They are based on a unique transformation key applied to the entire map area. The coefficients of these transformations are usually determined by adjustment by elements using the least squares method. The advantage of this method is to determine the most probable values of the coefficients including their characteristics of accuracy. According to the transformation equations we can find the following most commonly used methods: linear 2D transformations, polynomial transformations.

2D linear transformations are based on linear relations between the local coordinates on the old map and global coordinates in a defined coordinate system (identical points). In principle, these relations consist of set of simple geometric operations, resulting in a corresponding transformation. A map can be moved, rotated or its scale can be changed. Of course, these operations can be carried out independently in both coordinate axes. That results in the following transformation methods: similarity, 5-parameter affine, affine, and projective. The relevant transformation equations can be found in most works dealing with georeferencing. (Beineke 2001, Livieratos 2006)

Polynomial transformations, unlike the linear transformations, use a polynomial relation between the coordinates. Generally, in practice, the second and third order of polynomials is used (quadratic and cubic). Higher polynomials do not significantly improve the transformation method.

Besides the transformations that are defined by equations for the entire region, there is the possibility to transform the coordinates using methods, where each point corresponds to a separate mathematical relation. These methods can be classified as local, because the transformation key changes throughout the entire area.

1.2. Local Transformation Methods

The principle of local transformation methods are relationships based on the position of the transformed point. Distance to identical points is very often used as a parameter. This is actually the analogy to the interpolation in the space between the identical points. Identical points then usually show zero residuals after the transformation. Transformations are non-residual, unlike the previous ones, where just residuals of identical points are used for least squares adjustment.

Global methods preserve the spatial relationships in the image by the transformation equations (depending on the method used) and local methods distort the image so that the identical points were non-residual. There is essential to cover the entire area with identical points with appropriate density for local methods. Especially areas outside the envelope of identical points can be much distorted. On the other hand for the old (incorrect) map local transformations can better fit the image to the current state (e.g. for layers overlap).

Local transformation methods are widely used in combination with global methods. In the first step global transformation coefficients are computed; in the second step residuals after the transformation are interpolated. Two of the best known and most widely used methods include: inverse distance weighted (IDW) (Shepard 1968), thin plate spline (TPS) (Bookstein 1989).

Local transformation methods can also include transformations that are calculated by the image segments. The most common way is to divide space into irregular triangles whose vertices consist of identical points. There is a certain analogy with modeling surfaces using TIN. If the space is divided into triangles, there is need to select the appropriate transformation model. The simplest and most common method is to use the linear model. In each triangle there are determined six unknown parameters of affine transformation using coordinates of three apex points. The advantage of this procedure is primarily in its simplicity. Properties of transformed areas correspond with affine transformation. At the intersection of two triangles the map drawing is continuous (property of affine transformation), but lines are not smooth. This is due to the fact that the applied transformation surface is not smooth (it is general polyhedron). Besides using the linear model it is possible to transform triangular areas with smooth surfaces. In this case, we require the continuity of the transformation surfaces of individual triangles. There is necessary to ensure that the surfaces have always common tangent plane at the apex. The theory of these surfaces is beyond the scope of commonly used methods. More can be found in (Farin & Hansford 1977).

Another option is the division of the map by the quadrangles. Such nets are not much used in practice; however, they can have advantages. This method can be used in cases where we have identical points on the map frame. Then it is advantageous to divide the entire image by these frame marks. Each part of the image is then transformed separately and uneven map deformation is removed. Use of such method is to remove the map sheets' shrinkage. Projective transformation is used as a transformation method. It fits precisely using four identical points (corners). Use of projective transformation is the disadvantage of this method, where map drawing at the intersection of two quadrangles after transformation can be discontinuous. Most of map deformations are not so big that the discontinuity is not noticeable, but sometimes it can occur. Then it is necessary to look for other solutions using local non-residual transformations.

2. Map series georeferencing

When georeferencing maps, we can easily encounter situations where we are georeferencing two or more adjoining map sheets. Then it is desirable that the edges of adjacent map sheets exactly match. There are several possibilities how to ensure the continuity. The basic method is to merge the whole image before georeferencing. It can be used with a small number of map sheets and relatively good quality of image data. Using graphical software the image data can be combined so that they fit exactly on the edges. The large amount of resulting data (practically unrealistic is to merge a larger number of map sheets) is limiting this method. Other problems can be need of good knowledge of graphic software for the quality result or the inability to simply merge data (map sheets) with different coordinate systems.

Another option has two steps. In the first step the coordinates of the corners of map sheets are determined and in the second step the projective transformation using these corner coordinates is performed. Projective transformation is advantageous in terms of transforming just four identical points (corners) when it becomes non-residual. First, it is necessary to determine the coordinates of the corners of map sheets. This can be done either directly (from the knowledge of map sheet frame) or indirectly from identical points. Separate transformation of each map sheet can get the different coordinates of the same corner points. One corner point might correspond to a total of four adjacent map data sheets, and thus may also be determined four times. Then, the resulting coordinates are determined as an average. The averaged coordinates are then possible to be used within projective transformation.

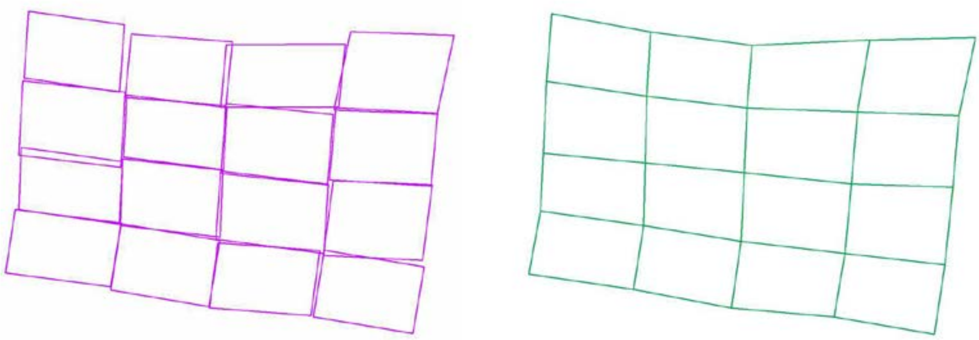

Figure 1. Map sheets after separate affine transformations (left) and after following averaging of corners and projective transformations (right).

As mentioned, the use of projective transformation is the disadvantage of this method, where map drawing at the intersection of two quadrangles after transformation can be discontinuous. Two step transformation, which can be time consuming, can be another problem. It is also not possible to transform the map with curved edges (only a linear shape of the frame is possible). Relatively easy procedure without a deeper knowledge of graphic software or a more complex adjustment could be the main advantage. Another method that will be described in more detail and presents newly proposed solution is overall adjustment of transformation coefficients of all map sheets with the conditions that define the edges continuity. The method works with identical points of all map sheets and with conditions defining edges connectivity. Here a calculation of all the coefficients is performed simultaneously. The image data are then transformed separately according to determined coefficients. This eliminates the dual transformation of image data and working with huge data. Unfortunately, there is need for a new adjustment when changing any identical point, because everything is computed together.

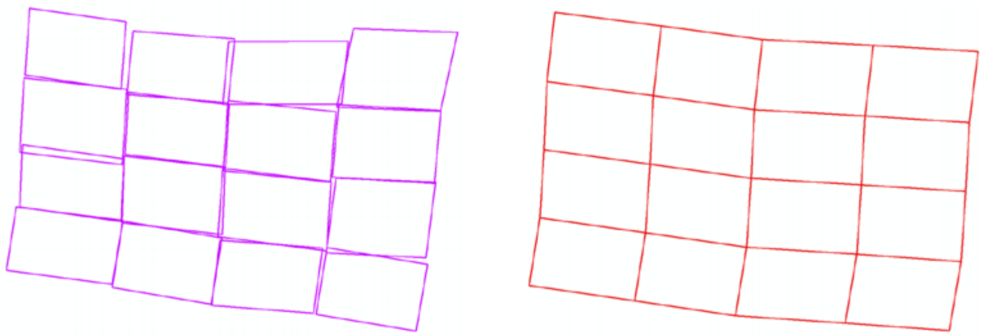

Figure 2. Map sheets after separate affine transformations (left) and after new proposed method with overall adjustment (right).

A more complex method based on surfaces geometry is using parametric surface patches. It is possible to define an area of patch using boundary curves (map sheet edges). It is necessary to obtain identical points on these edges. Then it is possible to reconstruct a new patch shape. This method is applicable to maps with frame markers, through which you can get good shape of boundary curves. (Coons 1977)

2.1. Affine transformation with edges continuity

As a first example of overall adjustment of transformation coefficients there are introduced relations for the affine transformation, which is probably the most widely used method. For simplicity, the selected base case consists of the transformation of two adjacent map sheets. Left map sheet has number 1 and the other (right) map sheet has number 2.

For sheet No. 1 we have transformation:

$$\begin{aligned} x' &= a_1 x + b_1 y + X_{t1} \, , \\ y' &= c_1 x + d_1 y + Y_{t1} \, . \end{aligned}$$

For sheet No. 2 we have transformation:

$$\begin{aligned} x' &= a_2 x + b_2 y + X_{t2} \, , \\ y' &= c_2 x + d_2 y + Y_{t2} \, . \end{aligned}$$

Conditions of identical shape of both common edges are based on the identity of two common corner points (ends of the edge). Thus:

$$\begin{aligned} {}^1x'_{RT} &= {}^2x'_{LT} \, , \\ {}^1y'_{RT} &= {}^2y'_{LT} \, , \\ {}^1x'_{RB} &= {}^2x'_{LB} \, , \\ {}^1y'_{RB} &= {}^2y'_{LB} \, , \end{aligned}$$

where the coefficients 1,2 are map sheets numbers, indices L, R represent the left, respectively right corner and indices T, B are top, respectively bottom corners of the map sheet. Substituting the transformation equations we get:

$$
\begin{aligned}
a_1{}^1x_{RT} + b_1{}^1y_{RT} + X_{t1} &= a_2{}^2x_{LT} + b_2{}^2y_{LT} + X_{t2}\,,\\
c_1{}^1x_{RT} + d_1{}^1y_{RT} + Y_{t1} &= c_2{}^2x_{LT} + d_2{}^2y_{LT} + Y_{t2}\,,\\
a_1{}^1x_{RB} + b_1{}^1y_{RB} + X_{t1} &= a_2{}^2x_{LB} + b_2{}^2y_{LB} + X_{t2}\,,\\
c_1{}^1x_{RB} + d_1{}^1y_{RB} + Y_{t1} &= c_2{}^2x_{LB} + d_2{}^2y_{LB} + Y_{t2}\,.
\end{aligned}
$$

Condition equations after small change are:

$$
\begin{aligned}
a_1{}^1x_{RT} + b_1{}^1y_{RT} + X_{t1} - a_2{}^2x_{LT} - b_2{}^2y_{LT} - X_{t2} &= 0\,,\\
c_1{}^1x_{RT} + d_1{}^1y_{RT} + Y_{t1} - c_2{}^2x_{LT} - d_2{}^2y_{LT} - Y_{t2} &= 0\,,\\
a_1{}^1x_{RB} + b_1{}^1y_{RB} + X_{t1} - a_2{}^2x_{LB} - b_2{}^2y_{LB} - X_{t2} &= 0\,,\\
c_1{}^1x_{RB} + d_1{}^1y_{RB} + Y_{t1} - c_2{}^2x_{LB} - d_2{}^2y_{LB} - Y_{t2} &= 0\,.
\end{aligned}
$$

Now classic method of adjustment by elements with constraints can be used:

$$
\begin{pmatrix} \mathbf{x} \\ \mathbf{k} \end{pmatrix} = \begin{pmatrix} \mathbf{A}^T\mathbf{A} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{A}^T\mathbf{l} \\ \mathbf{0} \end{pmatrix},
$$

Matrix **A** consists of affine transformation derivatives:

$$
\mathbf{A} = \begin{pmatrix}
{}^1x_1 & {}^1y_1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
{}^1x_n & {}^1y_n & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & {}^1x_1 & {}^1y_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & {}^1x_n & {}^1y_n & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & {}^2x_1 & {}^2y_1 & 0 & 0 & 1 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & 0 & 0 & {}^2x_m & {}^2y_m & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & {}^2x_1 & {}^2y_1 & 0 & 1\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & {}^2x_m & {}^2y_m & 0 & 1
\end{pmatrix}.
$$

Matrix **B** consists of condition equations derivatives:

$$\mathbf{B} = \begin{pmatrix} {}^1x_{RT} & {}^1y_{RT} & 0 & 0 & 1 & 0 & -{}^2x_{LT} & -{}^2y_{LT} & 0 & 0 & -1 & 0 \\ 0 & 0 & {}^1x_{RT} & {}^1y_{RT} & 0 & 1 & 0 & 0 & -{}^2x_{LT} & -{}^2y_{LT} & 0 & -1 \\ {}^1x_{RB} & {}^1y_{RB} & 0 & 0 & 1 & 0 & -{}^2x_{LB} & -{}^2y_{LB} & 0 & 0 & -1 & 0 \\ 0 & 0 & {}^1x_{RB} & {}^1y_{RB} & 0 & 1 & 0 & 0 & -{}^2x_{LB} & -{}^2y_{LB} & 0 & -1 \end{pmatrix}$$

l is the vector composed of defined coordinates of the identical points in the target system. After a simple calculation we can determine all unknown transformation parameters for the two map sheets. Condition equations ensure that the corners fit perfectly after the transformation.

For the successful adjustment it is necessary to define only independent conditions. When adjusting multiple map sheets there must be previously think about which conditions should be used. In the particular case, for four map sheets touching at one point it is possible to use only 3 conditions of equality coordinates from a total of 6 possible combinations of pairs of coordinates (map sheets). The remaining conditions can no longer be used, because they are linear combination of three used conditions.

Transformation coefficients applied to the images of the map sheets ensure the identity of both shared corners of map sheets and map content is absolutely continuous. If we wanted eliminate distortion of the map frame (e.g. bended linear frame), it would be necessary to reconstruct the image shape before georeferencing. This can be achieved by the aforementioned method of surface patches.

If we want to use proposed method with a different (easier) 2D linear transformation instead of affine, one should bear in mind the geometric nature of this method. This means that before the computation all map sheets should be able to fit together after used transformations.

2.2. Polynomial transformation with edges continuity

A second example of overall adjustment of transformation coefficients can be the second order polynomial transformation. It was chosen mainly because of relatively frequent use of this method for georeferencing of old maps. Partly similar method proposes Molnar (2010). Unfortunately, his method works only with perfect rectangular maps, cropped by the bounding box parallel with coordinate axes. In fact, his method is special (simpler) case of our new general method.

For simplicity, a base case will be shown again (transformation of two adjacent map sheets). Left map sheet has number 1 and the other (right) map sheet has number 2.

For sheet No. 1 we have transformation:

$$
\begin{aligned}
x' &= a_1 x^2 + b_1 y^2 + c_1 xy + d_1 x + e_1 y + f_1 \,, \\
y' &= g_1 x^2 + h_1 y^2 + i_1 xy + j_1 x + k_1 y + l_1 \,.
\end{aligned}
$$

For sheet No. 2 we have transformation:

$$
\begin{aligned}
x' &= a_2 x^2 + b_2 y^2 + c_2 xy + d_2 x + e_2 y + f_2 \,, \\
y' &= g_2 x^2 + h_2 y^2 + i_2 xy + j_2 x + k_2 y + l_2 \,.
\end{aligned}
$$

In the case of using a polynomial transformation the identity of the corner points is not enough for ensuring continuity through the whole common edge. The general solution of the common edge identity is the identity of functional form of the edge curve. In practice, the identity of three points of the curve (for second polynomial order) after transformation is sufficient. Conditions of identical shape of both common edges are:

$$
\begin{aligned}
{}^1x'_{RT} &= {}^2x'_{LT} \,, \\
{}^1y'_{RT} &= {}^2y'_{LT} \,, \\
{}^1x'_{RB} &= {}^2x'_{LB} \,, \\
{}^1y'_{RB} &= {}^2y'_{LB} \,, \\
{}^1x'_{RM} &= {}^2x'_{LM} \,, \\
{}^1y'_{RM} &= {}^2y'_{LM} \,,
\end{aligned}
$$

where the coefficients 1,2 are map sheets numbers, indices L, R represent the left, respectively right edge and indices T,B,M are top, bottom, respectively middle points of the edge. Substituting the transformation equations we get 6 condition equations.

Method of adjustment has the same scheme as the affine method. In addition, matrices **A** and **B** have similar shape with polynomial transformation derivatives for **A** and condition equations derivatives for **B**. Of course the size of matrices is different due to different number of transformation parameters.

After calculation we can determine all unknown transformation parameters for the two map sheets. Condition equations ensure that all the edges fit perfectly after the transformation. Transformation coefficients applied to the images of the map sheets ensure that the map content is absolutely continuous.

3. Experiments and discussion

As the most suitable candidate for the use of the newly proposed method seem to be results of the first military mapping survey of Austria-Hungary (1763-1785). There were selected two testing areas where identical points were collected. After their statistical testing they entered into the calculation of affine transformation with edges continuity. For both areas the results are very promising. All map sheets are continuous and fit precisely. Detail of four adjoining sheets can be seen on the Figure 3.

The second method with using second order polynomial transformations will be tested in the near future. Another plan to the future is evaluating of proposed georeferencing methods precision. When using least squares method it is possible to compute standard errors for all determined parameters as well as for any identical point. Using this evaluation it is possible to describe quality of the georeferenced map sheets. First tests show that standard positional error of the whole produced map is two times bigger than standard positional error of the individual map sheet. Comparing both methods (affine and second order polynomials) show that using polynomial method reduces the positional error by half.

There seems to be a very interesting project of georeferencing of all map sheets of the first military mapping survey in the area of Czech Republic. It also seems possible to include proposed algorithms in the software Georeferencer (Pridal 2011) for map series georeferencing.

Figure 3. Detail of four neighboring map sheets after using the new proposed method of georeferencing.

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