J. G. Lehmann's system of slope hachures – an investigation on the quality of relief representation at the beginning of the 19th century

Wolf Günther Koch

Dresden University of Technology, Institute for Cartography, Germany

Abstract. J. G. Lehmann (1765–1811) created a mathematically well-founded slop hachure system that undoubtedly had a paradigmatic effect on the theory of cartographic relief presentation. Lehmann was providing an optimal solution for the typical relief conditions in central Germany. Advantages and disadvantages are clearly visible on visual inspection and cartometric measurement of Lehmann´s original maps. In addition to the Saxon hachured maps, the (first) French map sheets with Lehmann hachures are included. As with the Saxon maps, the French maps accurately represent the slope in 5° zones. The representation of the character of the landscape is cartographically correct. The representation of individual features (mountains, hills, valleys, etc.) is oversimplified in some cases.

Keywords: relief presentation, slop hachures, map use, cartometry

1. Significance and reception of Lehmann hachures; current research status

The occasion for focussing afresh on Johann Georg Lehmann and his work was the 200th anniversary of his death on 6 September 2011. Because his eventful life and the legacy of his work are generally known to map historians, I would not like to treat its biography here. The main source for all biographies published in the past 200 years, is the preface to G.A. Fischer's third edition of Lehmann's "Die Lehre der Situationszeichnung..." ("Art of Topographic Drawing..." (1828). The current Internet biography provides only the text of Siegmund Guenther from the *Allgemeine Deutsche Biographie* ("General German Biography") in 1883. Cmok (1971) also particularly researched his childhood and youth, determining among other things a

new birth date (11 March 1765). The short biography in the Lexicon of Cartography and Geomatics is from Stams (2002).

As you know, at the end of the 19th Century, Lehmann created a mathematically well-founded slope hachure system, which *undoubtedly* had a paradigmatic effect on the theory of the cartographic relief depiction. Despite initial harsh criticism from the Prussian side (Lecoq, Mueffling), the new system was adopted not only in Saxony, but also in Prussia (1816), France (1817) and soon thereafter in England, the Netherlands and Russia. In Poland, Lehmann probably used slope hachures for the first time in his plan of Warsaw (*Plan Miasta Warszawy*, recorded 1807 [or 1808?], and engraved in copper in Dresden, 1809). The Lehmann system was included in all relevant textbooks, handbooks and dictionaries of cartography over the last 200 years. Before being published by G.A. Fischer, his work "Die Lehre der Situationszeichnung..." was translated into French on the orders of Napoleon, then into English with the permission of the Duke of York. Lehmann published its system later in further books.

As already mentioned, Lehmann's contemporaries, and then especially the 19th-century military cartographers, held a somewhat critical view of his method of slope hachures (Papay 1998), but there was also much support from the outset. Since the 1840s, even the last sceptics were convinced and became followers of Lehmann. In the first half of the 19th Century, Lehmann's hachures were modified in Germany by F. Schneider, E.J. v. Humbert, F. Schienert, L. Lyncker and C.F. v. Mueffling. The aim was to improve the readability of the gradient (slope) and military usefulness. Lehmann's system proved its worth particularly in hilly terrain and low mountains. In Bavaria and Austria, the scale was extended to be appropriate for use in the mountains with slopes greater than 45°. The depiction of relief in large- and medium-scale topographic maps was dominated throughout the 19th century by the more-or-less strictly applied Lehmann system. It was increasingly displaced by the combination of contour lines and shading only after the First World War.

The beginning of scientific investigations into the Lehmann system in the 20th century can be attributed to Karl Peucker (1898); Max Eckert (1921) also discussed and evaluated the Lehmann method in detail in the first volume of his "Die Kartenwissenschaft" ("Science of Maps"). With respect to the accuracy of the representation, he mentions the magnification (vertical exaggeration) of real relief forms. This approach results in a more sculptur-

al (three-dimensional) effect. This peculiarity of the presentation had already been described and commented on by Chauvin (1854). Eckert did not undertake detailed individual studies. Sydow (1863) referred to problems in the representation of depressions (negative forms of relief). Subsequent works until the publication of Eduard Imhof's fundamental work "Cartographic Relief Presentation" (1982) were largely descriptive in character. Imhof performed a precise and detailed analysis of the Lehmann system. He expressed his opinion on the hachure density and its influence on the graphic style of the map. Regular hachures require a simplified and smoothed terrain. The degree of generalization is much greater for a hachure map than for a contour map on the same scale. Imhof points out that hachure maps are based on very inaccurate and rough topographic mapping. He criticizes the distortion of features. Measures of the corruption and accuracy of elevations and slopes are lacking. Studies of accuracy are also missing from two German dissertations. These investigations (Cmok 1971 and Lindemann 2005), although very thorough in principle, also contain no analysis of accuracy, so there is no comparison between Lehmann´s hachure maps and the highly accurate topographic maps of today. Then, for the first time, W. Cmok (1971) at the Dresden University of Technology attempted to analyse and evaluate Lehmann's contributions to cartography beyond the issue of representation of relief. Cmok also cast some light on the economic and cultural/scientific-historical background in the late-18th and early-19th centuries. This showed that Lehman was active not only in topography and cartography, but also in a complex geo-scientific environment.

2. Lehmann's system of slope hachures as a novel representation of relief

Hachures were already used to portray relief in maps a long time before Lehmann. They were usually drawn as simple shading hachures to achieve a sculptural effect. Sketched hachures were used in the first modern national mapping works (the Cassini Map of France, the Josephine Austro-Hungarian land survey, the Saxonian original “mile sheets”). Acute-angled intersections (cross-hatching) created shading that illustrated steep terrain slopes (see the ordnance survey maps of Saxony). These representations were highly subjective. Even Lehmann was still using cross-hatching for

steep slopes in his early mapping before developing and then consistently applying his own mathematically based system.

Previously, Prussian Engineer-Major Ludwig Mueller had suggested depicting the greater or lesser steepness of slopes using vertical hachures according to the principle "the steeper, the darker". Indeed, Mueller's approach spread rapidly though the military schools, but the levels were characterized only verbally and thus open to subjective interpretation. Arbitrariness had free reign.

Lehmann's basic ideas:

- "The steeper, the darker" – assuming vertical illumination of the landscape (not strictly implemented).
- The quantity of incoming light depends on the angle of inclination of the surface (cosine of the slope angle).
- Since for 45° the slope is already completely dark: use of a linear function – degree of darkening is proportional to the slope angle.
- The hachures are set perpendicular to the contour lines.
- The hachure density (lines per inch) is mainly dependent on the scale of the map.

"For every 0–45° inclination of a plane expressed by lines (hachures), the ratio of the amount of black to the amount of white corresponds to ratio of the given slope angle to its complementary angle at 45°, or to the doubled tilt angle to its complement at 90°."

By limiting the scale to the range between 0° and 45° and using steps of 5 degrees, readability was significantly improved compared to earlier representations. Since the hachures are also slope lines (plan projections of profile sections) between the contour lines (not shown), the slope of the terrain and the (projection of) the direction of fall are readable at any point in the map. The absolute heights have to be represented by additional data points. The hachures of adjacent rows are slightly offset from each other to better illustrate sloping zones.

Lehmann was developing a scientific and mathematical basis for the slope hachure system. He was also providing an optimal solution in relation to

the typical relief conditions in central Germany (valleys cutting through plateau and hilly landscapes). At that time his new method was considered to be ideal for the military function of topographic maps and the technical options for reproducing maps (Papay 2002). The new method was also associated with a form of standardisation. Ideal test areas were then the geomorphologically striking ranges of hills and valleys near Dresden, in particular the “Plauensche Grund” and the areas surrounding the town of Tharandt.

Particular disadvantages were the relative darkening of the map image in locations of steep relief and the very high workload. The method was not suitable for relief forms of higher mountains. There were problems in correctly depicting a typical relief even on small scales.

3. Quality of relief representation in Lehmann hachured maps

Some characteristics of hachure representation have already been mentioned. Advantages and disadvantages are clearly visible on visual observation and evaluation of Lehmann's original maps.

Lehmann hachure maps (current state) can be compared with modern contoured maps (target state). This was the main method used by the author in his investigations. The cartometric study was carried out at six test areas near the city of Dresden. An example of a test area is shown in Figure 1.

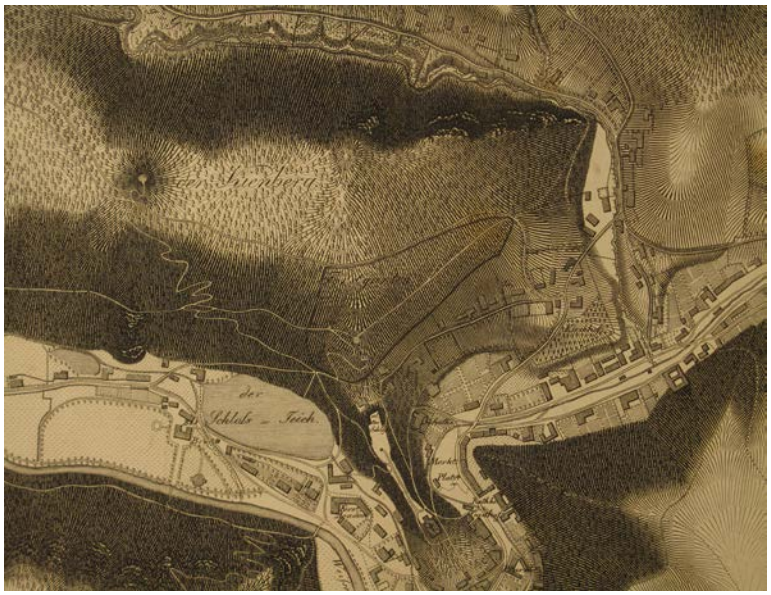

Figure 1. „Plan v. Tharand und der Umgegend (map extract), aufgenommen und gezeichnet von J. G. Lehmann, gest. von Bach, Dresden 1818“, map scale (O.) ca. 1: 4000. (Courtesy of SLUB Dresden)

Principal results of the study of German hachured maps by Lehmann (copperplate prints):

- High accuracy was achieved in the topographic mapping and graphical representation of the slopes (inclination). The inclination zones (5° zones) were largely true to reality.
- The general character of the landscape is reproduced in the presentation of the morphology (all features).
- However, the presentation of the individual features (mountains, hills, valleys, saddles, slopes, plateaus, etc.) is oversimplified in some cases. Asymmetrical forms are sometimes shown symmetrically, for example. Also, the geographical orientation of elongated features and groups of features is often inaccurate

There were also investigations into the first French maps drawn with Lehmann hachures. These historical maps (of 1813) were rediscovered only in 2010. An example of a test area is shown in Figure 2.

This four related map sheets of the area around Dresden with a 1:30,000 scale were published in Paris in 1849. They show the area of manoeuvres of the Prussian and Austrian armies in late August 1813, shortly before the Battle of the Nations near Leipzig. A comparison of the maps with the contents of the Saxonian mile sheets shows that these formed the basis for the French maps. Strikingly, however, a transformation of the hachures (parallel and cross-hatched) of the mile sheets into Lehmann slope hachures occurred. The reworking has been acknowledged as a cartographic masterpiece (Brunner, 2011). Initial features of the modification could be identified using domestic map comparisons, cartometric measurements and by inspecting the location. In many cases it became clear that the French depiction of the relief and inclines was done with remarkable precision. Of course, the historical possibilities of topographical mapping need to be considered. Also, it should be noted that the representation is based on the principles of Lehmann's theory.

It should further be taken into consideration that the new hachure depiction was done to a much smaller scale. This generalization has been don

satisfactorily both graphically and in terms of the function of maps. However, a certain schematization can be seen in the representation of the hilly

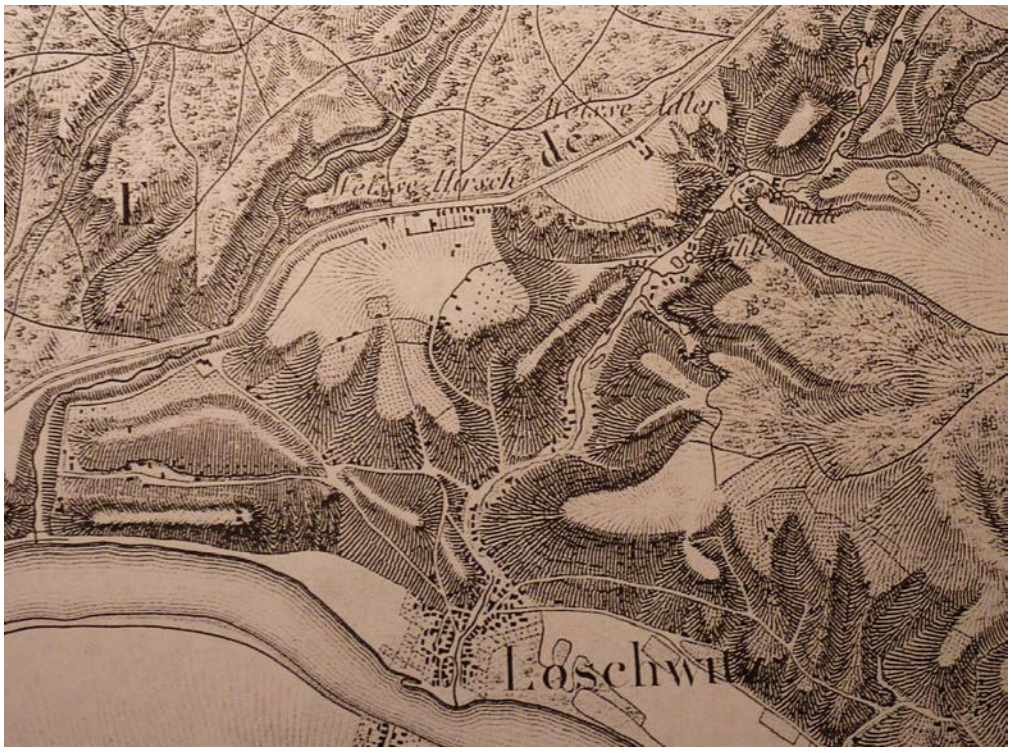

Figure 2. "Plan de Bataille de Dresde 26 et 27 Août 1813 – Théâtre des Manœvres, Paris 1849", (map extract „Loschwitz"), map scale (Orig.) 1:30.000. (Courtesy of SLUB Dresden)

landscape north of Dresden. In the future, accurate information on producing the new hachures can certainly be obtained only from French military archives.

Key results of study of the French hachure maps (copperplate prints):

- As with the German maps, the slope/inclination is correctly represented by 5° zones.
- The representation of the character of the landscape is cartographically correct (based on Saxon "Mile Sheets").

- Individual features (mountains, valleys, ridges, etc.) are largely true to nature. The cartographic generalization from 1:12.000 to 1:30.000 is satisfactory for the military use of the maps.

- The hachure density in these French maps on the 1:30.000 scale is 15 lines/cm in shallow-sloped areas and approximately 30 lines/cm in the steeper areas. The intensification or darkening is achieved by the closer positioning of the lines (hachures). These lines are of equal width. Lehmann also worked with different hachure densities, but his original theory was based on a fixed hachure density and variable hachure width.

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