Symbol Considerations for Bivariate Thematic Maps

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Abstract. Bivariate thematic maps are powerful tools, making visible spatial associations between two geographic phenomena. But bivariate thematic maps are inherently more visually complex than a univariate map, a source of frustration for both map creators and map readers. Despite a variety of visual solutions for bivariate mapping, there exists few empirically based 'best practices' for selecting or implementing an appropriate bivariate map type for a given scenario.

This research reports on a controlled experiment informed by the theory of selective attention, a concept describing the human capacity to ignore unwanted stimuli, and focus specifically to the information desired. The goal of this research was to examine if and how the perceptual characteristics of bivariate map types impact the ability of map readers to extract information from different bivariate map types.

55 participants completed a controlled experiment in which they had to answer close ended questions using bivariate maps. The results of this experiment suggest that 1) despite longstanding hesitations regarding the utility of bivariate maps, participants were largely successful in extracting information from most if not all of the tested map types, and 2) the eight map types expressed unique differences in their support for different map reading tasks. Though selective attention theory could explain some of these differences, the performance of the map types differed appreciably from similar studies that examined these map symbols in a more abstract, perception-focused setting. While the perceptual models of selective attention can still be useful in guiding map design, more work is needed to understand the cognitive aspects and limitations to bivariate map reading.

Keywords: Bivariate Maps, Map Perception, Thematic Mapping, Selective Attention, User Studies

Introduction

Existing Thematic Cartography literature primarily focuses on univariate maps, or cartographic representations that portray only one attribute of a geographic information set. Displaying two or more attributes (a bivariate and multivariate map, respectively) is a powerful way to convey information about associated geographic phenomena, but successfully designing bivariate/multivariate maps is challenging due to the added density of information.

The functional purpose of a bivariate map is to show relationships among two geographic phenomena (Fisher, 1982; Tyner, 2010). Visualizing this geographic relationship frequently provides insight into understanding the mapped phenomena. The cost of bivariate mapping is in comprehensibility. A bivariate map is, by its nature, more visually complex than a univariate map. Visual complexity makes the map more difficult for the viewer to process mentally, and, if this complexity proves overwhelming, it can render the map valueless to the reader. Fisher (1982: 268) puts the multivariate map in simple cost/benefit terms: there is "a limit beyond which the difficulty of comprehending two or more subjects exceeds the value of being able to relate them".

This writing supposes, along with Carswell & Wickens (1990) and Nelson (1999; 2000) that the challenges of designing and reading bivariate maps can be resolved, at least in part, by a better understanding of the perceptual and functional aspects of various bivariate mapping solutions. The parcity of user-based, comparative testing of bivariate map types presents a significant problem for the thematic map designer; with little theoretical or empirical basis from which to draw, the designer cannot make an informed decision on which bivariate symbolization scheme to use. There are many more design possibilities for bivariate mapping compared to univariate mapping; numerous bivariate map types are already in standard use, and new solutions continue to be proposed. Given this, the need to understand the capabilities and limitations of these various bivariate map types becomes all the more pressing.

Literature Review

2.1 Taxonomy of Bivariate Map Types

Cartography textbooks discuss bivariate and multivariate thematic mapping less systematically than, for example, univariate map types, map projections, or color schemes. An examination of different thematic cartography textbooks (Dent, Torguson, & Hodler 2009; Fisher, 1982; Krygier & Wood, 2011; Slocum et al., 2003; Tyner, 2010) reveals that these sources identify eleven different bivariate map types, which represent only a small selection of map types identifiable from published maps or the cartographic literature. No text describes more than four different bivariate map types, and only two map types (bivariate choropleths and choropleth with overlaid graduated symbols) are considered by more than two sources.

In the interest of creating a more exhaustive taxonomy of bivariate map types, this writing considers all bivariate map solutions to be a combination of two visual variables (size, shape, value, etc, as initially described by Bertin, 1967|1983) and two symbol dimensionalities (point, line, polygon, etc) (*Figure 1*). This is similar to the combinatorial systems proposed by Stefan et al., 2007, and Nelson, 2000.

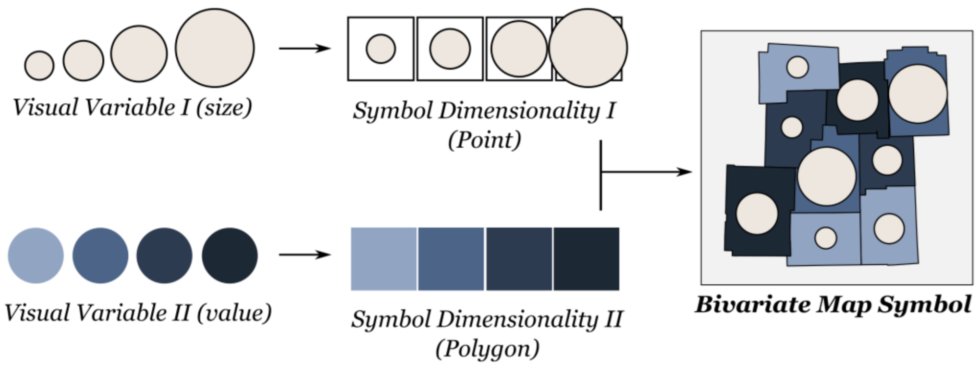
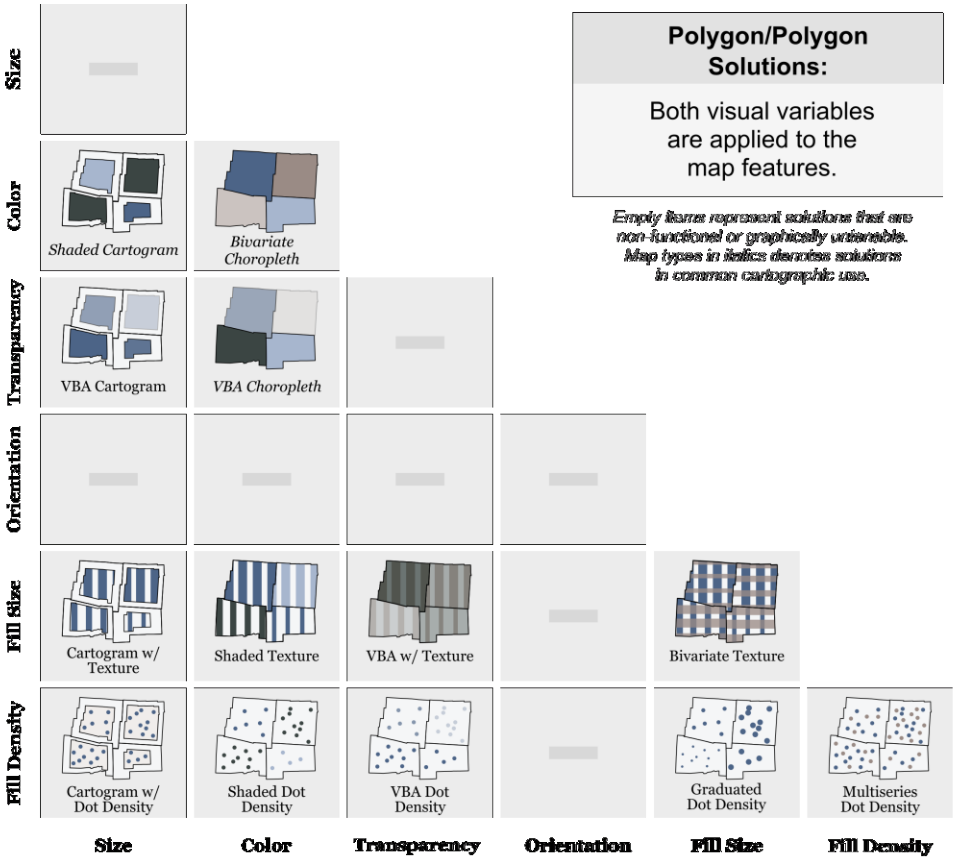
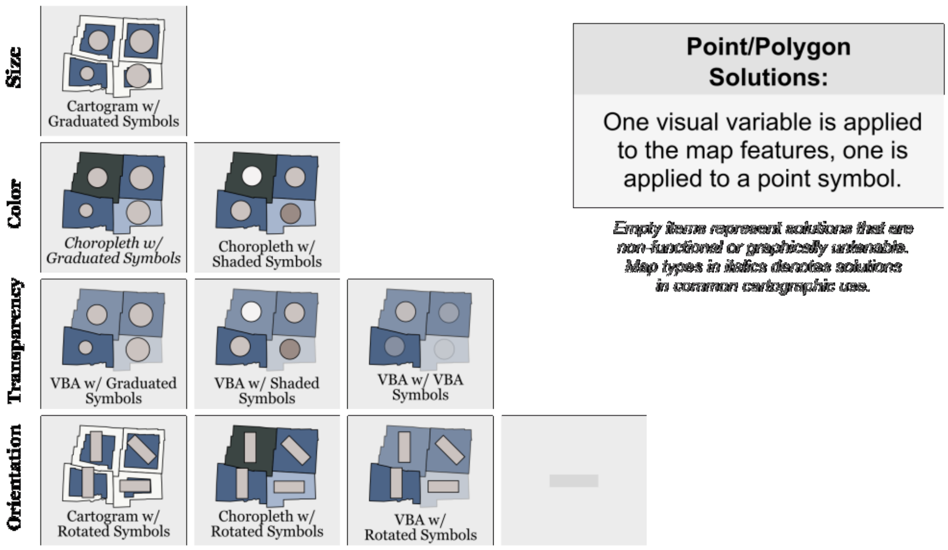


Figure 1. The constituents of a bivariate map symbol: two visual variables combined with two symbol dimensionalities.

Different authors have offered different collections of visual variables (Tyner, 2010), and these listings are sure to adapt as new visual variables are proposed. Depending on the number of visual variables considered, the number of two-variable combinations can range in the dozens to hundreds. Symbol dimensionalities, comparatively, are generally agreed to be limited to points (0d), lines (1d), and polygons (2d), with other authors also considering surfaces (2.5d) and volumes (3d) (Tyner, 2010, Stefan et al., 2007). The solution space for bivariate map symbols, then, is exponentially large. Fortunately, the number of tenable graphical solutions for a given use case can be minimized by considering the appropriateness of a given visual variable or symbol dimensionality to represent the data set (per MacEachren, 1995). As an example, *Figure 2* provides an example of graphic solutions appropriate to representing two sets of quantitative information enumerated to polygonal features.


Polygon/Polygon Solutions:
Both visual variables are applied to the map features.
Empty items represent solutions that are non-functional or graphically untenable. Map types in italics denotes solutions in common cartographic use.
Size
Color
Transparency
Orientation
Fill Size
Fill Density
Shaded Cartogram
Bivariate Choropleth
VBA Cartogram
VBA Choropleth
Cartogram w/ Texture
Shaded Texture
VBA w/ Texture
Bivariate Texture
Cartogram w/ Dot Density
Shaded Dot Density
VBA Dot Density
Graduated Dot Density
Multiseries Dot Density
Size
Color
Transparency
Orientation
Fill Size
Fill Density


Point/Polygon Solutions:
One visual variable is applied to the map features, one is applied to a point symbol.
Empty items represent solutions that are non-functional or graphically untenable. Map types in italics denotes solutions in common cartographic use.
Size
Color
Transparency
Orientation
Cartogram w/ Graduated Symbols
Choropleth w/ Graduated Symbols
Choropleth w/ Shaded Symbols
VBA w/ Graduated Symbols
VBA w/ Shaded Symbols
VBA w/ VBA Symbols
Cartogram w/ Rotated Symbols
Choropleth w/ Rotated Symbols
VBA w/ Rotated Symbols

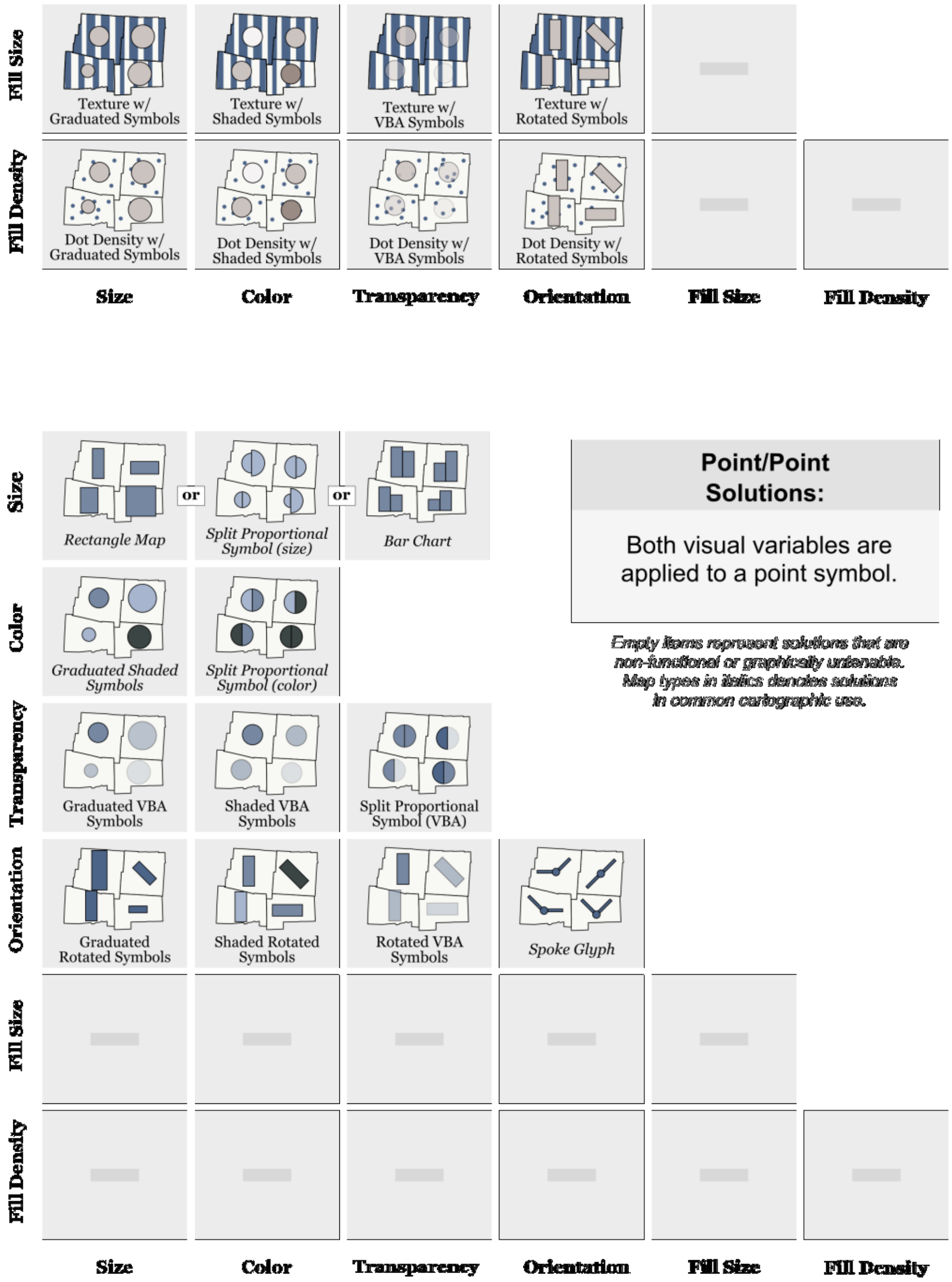


Figure 2. A taxonomy of bivariate symbol solutions appropriate for two sets of numerical data aggregated to a polygonal enumeration unit. Hue, saturation, and value have been combined into simply 'color' to avoid redundancy.

2.2 Bivariate Maps and Selective Attention

Defined simply, selective attention is the ability of an observer to attend to one stimulus while ignoring the confounding influence of others. In the context of complex visual stimuli (such as a map), selective attention manifests as the ability to attend to specific visual variables while ignoring the others. The theory of selective attention was developed by research psychologists, and has been proposed as a means to aid understanding and design of information graphics, including thematic maps (Carswell & Wickens, 1990; Shortridge, 1982; Nelson, 1999;2000). One contribution from Selective Attention research has been the concepts of separability, integrality, configurality, and asymmetry (ibid). These phenomena arise when two visual variables are combined in a symbol, describing the perceptual aspects of that visual combination.

In a separable combination, the viewer is capable of attending each individual visual variable, with minimal perceptual interference from the other. In an integral combination, the visual variables interact in such a way that attention to the individual variables are inhibited. In *Figure 3*, for instance, notice how the 'cluster' in the lower left is easier to notice as four squares in the separable combination, as opposed to four rectangles of equal width. Contrarily, it is easier to spot the rectangles with a large area as opposed to dark hexagons and pentagons.

Configurality exhibits properties of both separability and integrality, and tend to arise when the symbol employs the same visual variable twice. Uniquely, configural combinations have features that arise when the variables are 'in agreement', such as the unintereupted circles in *Figure 4*. Finally, asymmetrical combinations appear to represent unique or conditional interaction effects between the two visual variables. In *Figure 4*, for instance, variations in hue are least noticable when the symbols are very dark.

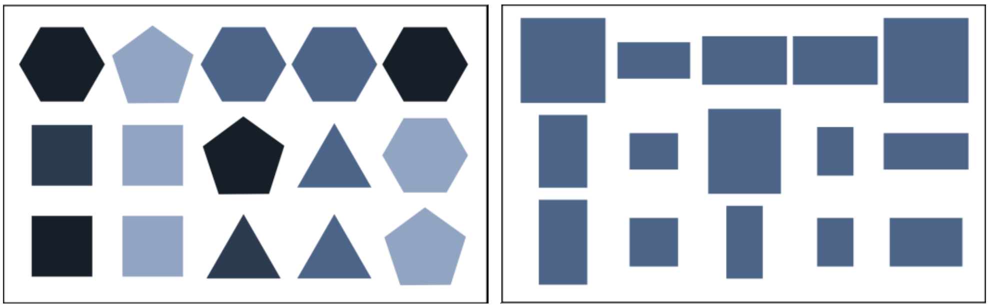


Figure 3. The same data represented using a separable combination (shape/value, left) and an integral combination (height/width, right).

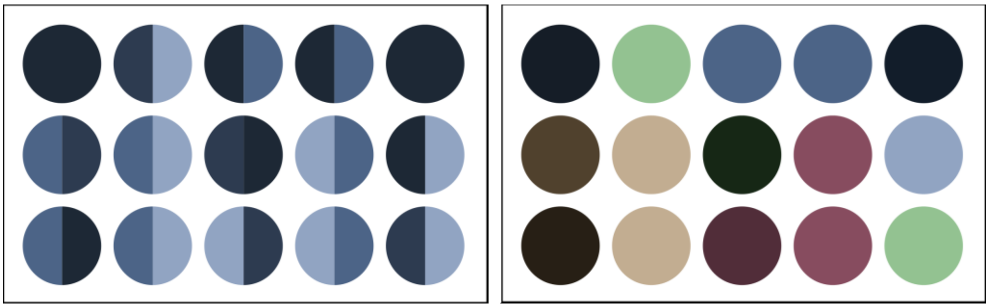


Figure 4. A configural combination (value/value, left) and an asymmetrical combination (hue/value, right).

2.3 Bivariate Maps & Encoding Information

In a bivariate map symbol, the two statistical attributes are encoded by the two visual variables used to construct the symbol. Each combination of visual variables also creates two emergent (gestalt) visual dimensions, which are visual variables unto themselves. A combination of height/width, for instance, creates two emergent visual dimensions: the total area of the symbol, and its directionality (vertical vs horizontal).

These gestalt visual variables encode information in the same way the original visual variables do. One encodes a positive association between the data (i.e., map features where both attributes are low vs features where both attributes are high) while the other gestalt dimension encodes negative association (features where one data variable is higher than the other, or vice versa). Given a matrix of symbols (as would be seen in a two-axis map legend), these emergent visual variables can be found along the orthogonal axes (*Figure 5*). (These orthogonal data axes will be referred to as the Plus(+) and Minus(-) axes, and the original information axes as the X and Y axes).

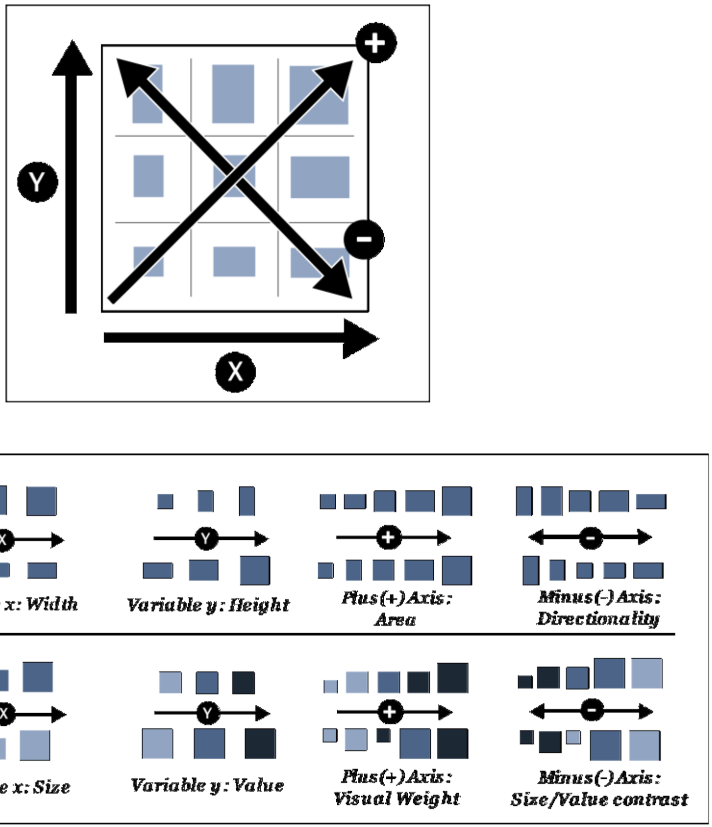


Figure 5. The orthogonal data axes (above), and two examples of the gestalt visual variables created within a bivariate map symbol.

Methods

A controlled experiment was administered to assess the variation in performance across various bivariate map design solutions. The purpose of the study was to examine empirically how different map types and different conditions of selectivity support different map reading tasks, and if this level of support varies according to differences in the expertise of the map reader. Conceptually, the experiment was designed to reconcile, at least in part, the methods employed in Selective Attention research and the methods employed in cartographic performance testing. This involved asking participants to perform a selection of map reading tasks across eight different bivariate map types, recording their accuracy and response time to each question. The survey opened with questions designed to assess each participants' expertise with regards to map use, and closed with questions on the users' personal preferences of the different map types, modified from questions used by Olson (1982).

A total of 55 participants participated in the study. The majority of participants were recruited from the Geography Department of the University of Wisconsin – Madison, although the study was open to any interested participants. Subjects were recruited purposefully to represent a range of experience and knowledge of cartography and spatial analysis. The study participants were offered 5 USD as compensation for their time.

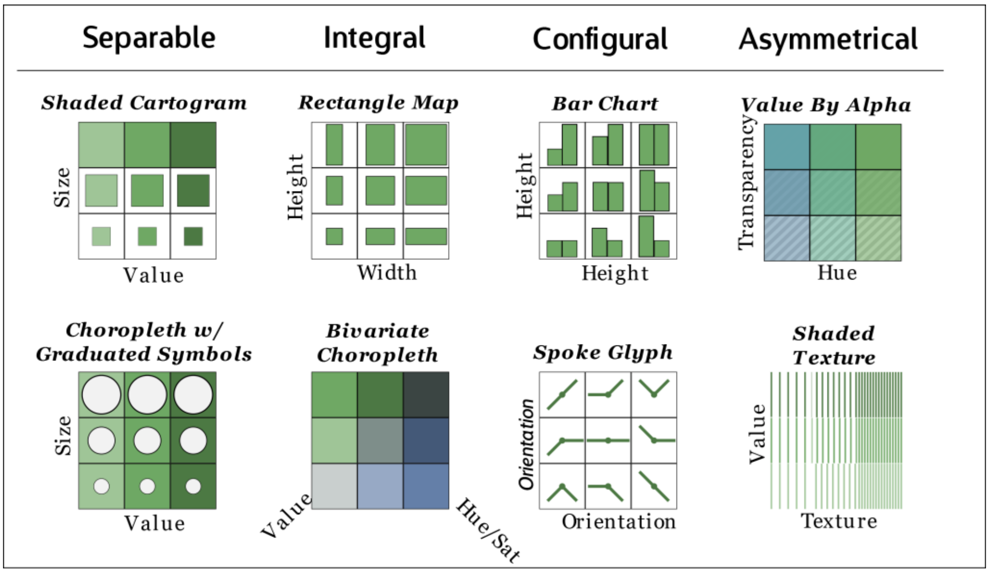


Figure 6. Legends for the eight map types tested.

Eight common bivariate map types, drawn from the map types discussed in *Figure 2*, were designed for inclusion in the experiment (*Figure 6*). Two map types were included for each condition of selectivity (separable, integral, configural, and asymmetrical). The map types were selected based on the following criteria: 1) if possible, there was existing research establishing the visual combination as separable, integral, etc., 2) the map type was particularly representative of the perceptual features associated with that selectivity, and 3) the map type is commonly used in cartographic practice.

Note that the design for the bivariate choropleth uses a modified construction from the other legend designs: the visual variables are applied across the orthogonal information axes, and it employs small variations in saturation as well as hue. This was done to align the legend design with established recommendations for the design of bivariate choropleth legends (Trumbo, 1981; Dunn 1989). The design for the value-by-alpha legend included a pattern fill to allow for a better depiction of transparency; on a matte white or black background, variations in transparency are indistinguishable from variations in value, rendering a value-by-alpha map conceptually identical to a bivariate choropleth map (Roth, Woodruff, & Johnson 2010).

Each map legend was applied to a basemap composed of 36 counties from western Ohio, rotated 90 degrees, and modified slightly in size and topology. The allotment of data values within each map was normalized and randomized. Each map included a legend depicting the fictitious variables of 'pizza consumption' and 'chicken consumption' along an ordinal scale (high/medium/low) (*Figure 7*).

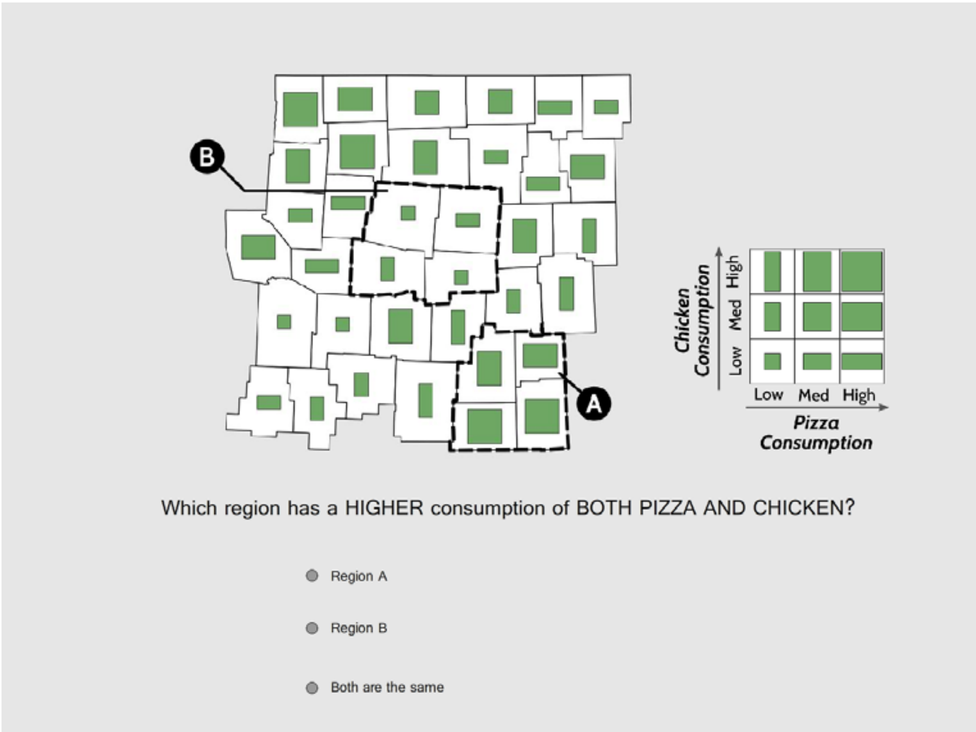


Figure 7. Screenshot from the test interface.

Each map type had eight accompanying questions, requiring participants to compare the values of map features across two levels of reading across and four search axes. The levels of reading included *Elementary* (looking at single map units) and *General* (looking at regions composed of multiple units). The search axes included the information axes described in *Section 2.3*: *X* (the first mapped attribute), *Y* (the second mapped attribute), *Plus(+)* (positive association between attributes; High/High to Low/Low), and *Minus(-)* (negative association between attributes; High/Low to Low/High). Each of these 'blocks' of eight questions opened with a screen that permitted participants to examine the example map and legend. The order of map types shown to participants, as well as the order of questions within each block, was randomized.

Results

3.1 Statistical Results

Participants were consistently successful in accurately answering the 64 questions (*Table 1*). The participants' global accuracy rate was 96.1%, and 26% of the map questions were answered correctly by every single participant. ANOVA analysis of accuracy rates between trials did not show statistically significant differences between accuracy across the different map types (p-value = 0.064) nor across the different conditions of selectivity (p-value = 0.0598).

Response times (RTs) showed statistically significant differences across both map type, map selectivity, and task ($P < 2e\text{-}16$, using ANOVA) (*Figure 8*). These differences indicate that the choice of map type has important impacts on how intuitively users are able to extract various forms of information from the map. The best performing maps (that is, those with the lowest average RT combining all tasks) were the rectangle map (21.15 secs) and the choropleth with graduated symbols (22.97 secs). The worst performers were the value-by-alpha (29.39 secs) and spoke glyph (38.33 secs).

	Shaded Cartogram	Choropleth w/Grad. Symbols	Bivariate Choropleth	Rectangle Map
X, Elementary	0.95	0.98	0.98	0.96
Y, Elementary	0.91	1.00	1.00	0.98
+, Elementary	1.00	1.00	1.00	1.00
-, Elementary	0.93	0.93	0.95	0.98
X, General	0.95	0.95	1.00	0.98
Y, General	0.96	0.98	1.00	0.98
+, General	0.98	0.95	1.00	1.00
-, General	0.93	1.00	1.00	0.95
	0.95	0.97	0.99	0.98

	Value by Alpha	Shaded Texture	Spoke Glyph	Bar Chart	
X, Elementary	0.96	0.81	0.93	0.96	0.942
Y, Elementary	0.95	0.89	0.96	0.96	0.957
+, Elementary	0.98	0.98	1.00	1.00	0.995
-, Elementary	0.98	0.96	1.00	0.95	0.959
X, General	0.93	0.98	0.85	0.91	0.943
Y, General	0.98	0.96	0.96	0.98	0.977
+, General	0.98	1.00	0.95	1.00	0.982
-, General	0.96	0.91	0.85	0.96	0.945
	0.97	0.94	0.94	0.97	**Averages**

Table 1. Accuracy rates by map type and task. Table divided for legibility.

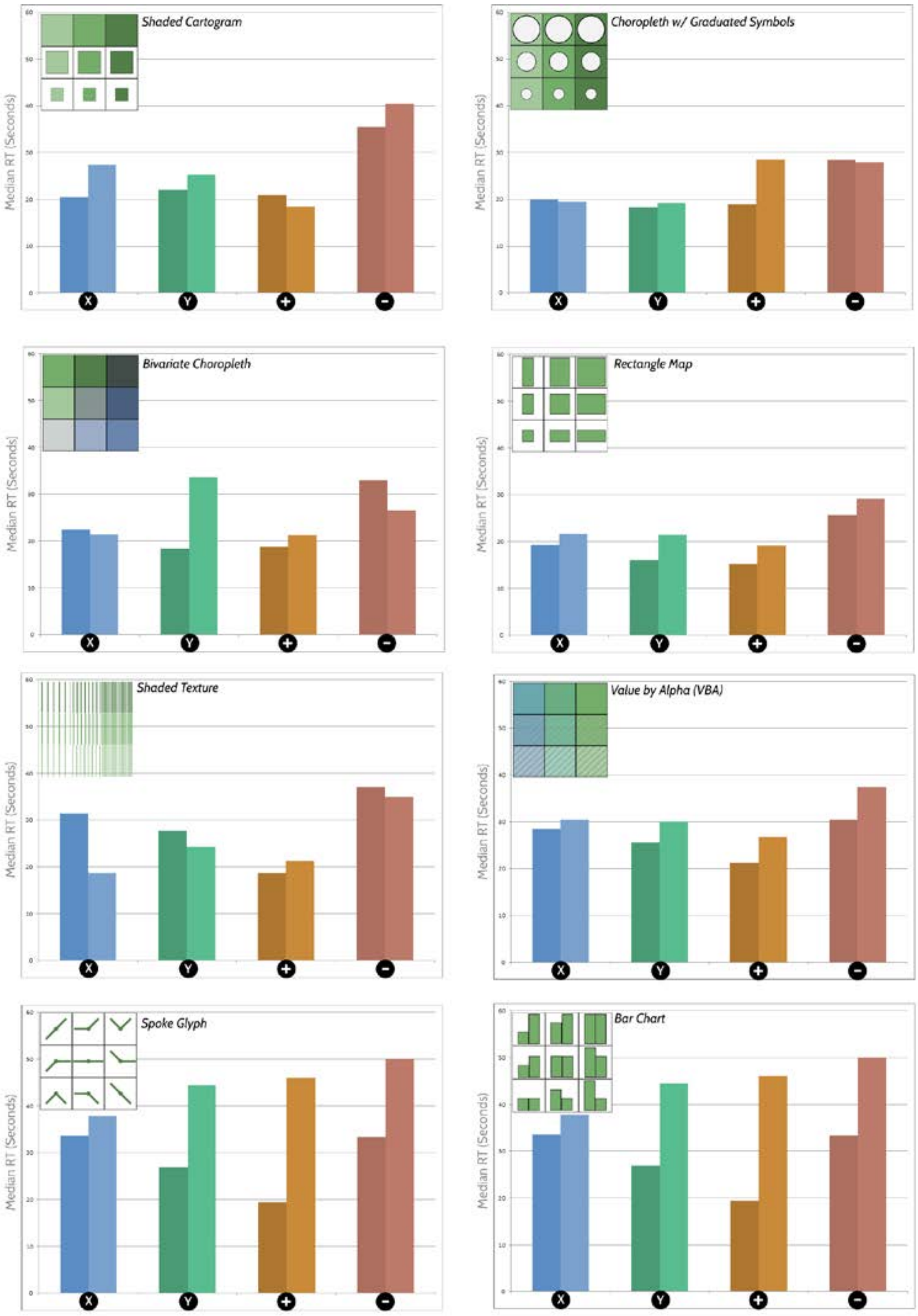


Figure 8. Reaction Times (RTs) by map type and task. Left side of bar represents elementary-level task, right side represents general level.

The closing questions on user preference showed clear differences among the eight map types (*Table 2*). Kruskal-Wallis testing (a non-parametric version of ANOVA) found statistically significant differences between how users ranked the map types (p value = 2.2e-16). *Table 3* provides a summary of the Likert scores. There is little variation in how participants responded to the questions within each map type, however: that is, if a participant liked or disliked a map type, they would rate it similarly high or low regardless of the question asked.

Participants' accuracy and response times did not significantly vary according to user expertise (*Table 3*). Participants reporting an educational background in cartography/GIS had an overall accuracy merely 0.04% better than those without. Nearly identical results occur using the other major measure of expertise, work experience. Again, no statistically significant differences were found between those with and those without work experience in Cartography/GIS.

	Shaded Cartogram		Choro. w/ Grad. Symb		Bivariate Choropleth		Rectangle Map	
	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev
Visually Displeasing <-> Visually Appealing	4.0	1.6	4.8	1.5	5.7	1.4	3.7	1.6
Usual <-> Unusual	4.5	1.5	3.6	1.7	2.5	1.7	4.6	1.6
Difficult to Read <-> Easy to Read	4.4	1.6	5.4	1.6	5.0	1.7	4.1	1.8
Does not Show Individual Distribution Clearly <-> Does " "	4.5	1.6	5.6	1.5	4.8	1.7	4.4	1.8
I Cannot Judge the Closeness of the Relationship <-> I Can " "	4.8	1.5	5.5	1.5	5.2	1.5	4.7	1.7
Bad overall <-> Good Overall	4.5	1.5	5.3	1.5	5.1	1.3	4.2	1.6

	Value by Alpha		Shaded Texture		Spoke Glyph		Bar Chart	
	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev
Visually Displeasing <-> Visually Appealing	5.3	1.6	2.4	1.6	2.4	1.4	3.7	1.4
Usual <-> Unusual	4.5	1.7	5.0	1.6	5.6	1.6	3.9	1.5
Difficult to Read <-> Easy to Read	3.9	1.6	2.5	1.4	2.5	1.7	4.3	1.9
Does not Show Individual Distribution Clearly <-> Does " "	3.8	1.7	2.9	1.7	3.4	2.1	4.9	1.7
I Cannot Judge the Closeness of the Relationship <-> I Can " "	3.9	1.5	3.2	1.7	3.7	1.9	5.0	1.6
Bad overall <-> Good Overall	4.0	1.4	2.6	1.5	2.7	1.6	4.3	1.6

Table 2. Likert scale results: mean and standard deviation for participants' scoring of the map types along various scales (all running 1-7, with 1 representing the descriptor on the left, and 7 the descriptor on the right).

Group	Average RT	Average Accuracy
Have taken coursework	33.1	0.955
Have not taken coursework	29.9	0.951
P-value	0.237	0.698

Group	Average RT	Average Accuracy
Has had work experience	33.2	0.951
Has not had work experience	30.5	0.956
P-value	0.296	0.613

Table 3. Global Response time (in seconds) and accuracy rates across cartography/GIS related user expertise.

3.2 Analysis

Despite hesitations about the utility of bivariate maps (described in *Section 1*), the participants in this survey were consistently successful in answering the questions presented to them. There were also several map types (such as the choropleth w/ graduated symbols and bivariate choropleth) that users rated largely positively in their capacity to read and understand the information on the map (based on the Likert scores). It should be recognized, however, that the structured, task-specific way participants interacted with the maps in this study would have engaged different visual and cognitive activities than the unstructured, spontaneous kinds of map reading that is more likely to occur in everyday life (Antes & Chang 1990).

Selective attention theory provided some insights into the performance of the eight map types, but could not account for all results seen. The best performance in the General Plus(+) task, for instance, was expected from the strong emergent dimensions of the integral map types, however the best performer in this task was the bar chart, a configural solution. The X and Y tasks were hypothesized to be best supported by the separable combinations, but this materialized in only one instance (the choropleth with graduated symbols, in the General Y task). Additionally, participants did not rate either separable or integral combinations as better at portraying individual distributions versus relational information. A given map type did not always perform identically to the other map type with the same selectivity; the disparities in performance between the configural solutions (bar chart and spoke glyph) provide a good example.

The eight map types showed frequent variations in reaction times across task, whether that be performing better than the other maps in one of the eight tasks, or particular map types showing variation in performance between Elementary and General level tasks within a given information axis

(*Figure 8*). There were also statistically significant differences between the eight tasks in terms of their accuracy rates. Speaking broadly, variations in task performance across the map types were relatable to the unique perceptual properties of their specific visual cues. In one case, the spoke glyph, the lack of strong visual cues made it challenging for participants to retrieve information regardless of task. The other map types were generally successful in supporting the eight tasks overall, but showed differences in performance dependent on the information axis or the level of reading. The configural map types, for instance, demonstrated a good ability for viewers to extract information along the Minus(-) axis, but only when attending to individual symbols (Elementary tasks); when challenged to perform the same task at the General level, participants seemed unable to visually aggregate the symbols into a similarly intuitive cue.

Conclusions

The major goals of this research were to 1) understand the unique perceptual qualities of different bivariate mapping solutions and 2) empirically test if these qualities impacted various forms of information retrieval. Although selective attention provided a useful framework to understand the perceptual characteristics of the map types, the results of the study differed from expectations derived from earlier research, which examined similar symbols in an abstracted setting.

This study assumed that the major challenges of reading a bivariate map were perceptual: that is, the visual variables interact with each other in such a way as to hinder information extraction from the map, and choice of map type could serve to minimize visual interference and maximize the intuitive representation of the information (or, at least, certain aspects of the information). But cognition appears to also be an important aspect of bivariate map reading. Other than the Likert scales, the information collected in the survey demonstrates very little about how the maps were cognitively understood by the participants. In order to understand why reaction times varied across task, for instance, it would have been beneficial to know the mental strategies the participants used to answer the questions. Did the participants begin by looking at the map, or at the question itself? How often, and in what contexts, did participants have to re-check the legend? Did they answer the General-level questions by attending to individual map features, or by trying to visually aggregate them? There are several methods that could better investigate such cognitive-based questions, such as eye tracking, focus groups, "think-aloud" experiments (Pickle 2003), or by giving participants more open-ended information-seeking challenges (rather than the highly specific information retrieval tasks used in this study).

Similarly, this research attempted to identify and discuss the emergent dimensions and features of various bivariate map types; dimensions such as the 'leftedness/rightedness' of a split proportional symbol, or the directionality (vertical/horizontal) of a rectangle map. The understanding of these various emergent visual dimensions is significantly less than the understanding of the more fundamental visual variables. Since these emergent dimensions do serve to encode information in a bivariate map, it would be enlightening to examine how these dimensions are perceived and their appropriateness for encoding different varieties of data.

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