The Application and Effects of Sky Models on Hill Shading

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Abstract. Sky models adopted by the International Commission on Illumination (CIE) define the radiance associated with all sectors of a hemisphere representing the sky dome and are used for illuminating three-dimensional objects and surfaces. Although commonly used in architectural "daylighting" studies and computer graphical renderings, cartographers more often utilize a simple point source illumination model. We present an application and methodology for hill shading terrain with various sky models. The desktop application efficiently samples sectors of various sky models and returns a number of points in the sky representing discrete illumination vectors. It also provides weights associated with the luminance of each sky sector to be used to illuminate the terrain. Sets of approximately 250 tuples (illumination vector azimuth and inclination, along with a weight) can be used to smoothly shade and softly shadow terrain.

We also look at the salient qualities of the resulting terrain maps. We classify sky models into directional and non-directional categories, with the latter having rotational symmetry with respect to the sky's zenith. Directional illumination models include the sharp, clear day, and turbid sky models. Like point source illumination, all of these are effective for obliquely illuminating terrain when the sectors of most intense radiance are positioned in the northwestern sky. In addition, sky models are able to reveal details of terrain not apparent with point source illumination. Non-directional models include the overcast and uniform sky models. Due to their sky symmetry, both result in shadings similar to slope shading. Important differences exist, however, between slope shading and hill shading with non-directional sky models. While slope shading has been summarized as "the steeper the darker", shading from overcast and uniform sky, and to a lesser extent clear and general sky, is better summarized as "the less of the sky

visible the darker". This effect is especially apparent in an incised terrain such as a canyon. Although many factors are at play in determining how much of the sky hemisphere is visible, models with radiance distributed throughout the sky result in hill shading in which elevation values correlate to shades of gray in different manners than the shades of gray associated with point source illumination.

Keywords: Hill shading, daylighting, sky models, slope shading

1. Introduction

The illumination model generally implemented in geographic information system software is the point source model. The point source model uses Lambert's Law of Cosines to establish a simple relationship between the incident angle of illumination and the brightness of the surface unit (Chang, 2011). This simplified model served as the de facto practice for hill shading for both hand-rendered (Imhof, 1982), and analytical cartography (Kimerling et al., 2011).

Numerous sky models have been developed and utilized other disciplines such as architectural studies and computer graphics. These models quantitatively define the distribution of sky luminance in a hemisphere (Nishita and Nakamae, 1974) that is then used to illuminate the terrain. We organize these sky models into ones with a "directional" component and those that are "non-directional." Non-directional sky models exhibit radial symmetry with respect to the hemisphere's zenith.

Directional sky models include sharp, clear day (Darula and Kittler, 2002), and turbid sky illumination (Darula and Kittler, 2002). Although the general direction of illumination can be from any sector of the sky, northwestern illumination is consistent with traditional hill shading techniques (e.g. Imhof, 1982; Slocum et al., 2008). Non-directional illumination includes uniform and overcast sky illumination (Moon and Spence, 1942; Nakamura et al., 1985). These renderings will not include cast shadows, and lack a perceptual relief effect (Moellering, 2012; Moellering, 2000).

Hill shaded maps resulting from various sky models appear to have much different distributions of shades of gray than those resulting from point source illumination. For example, we would expect a map rendered with slope shading as described by Imhof (1982) to have a very strong, positive correlation with slope values. In this study, we compare the grayscale values of maps hill shaded with various sky models to the primary terrain variables of elevation and the derived variables of slope and aspect to look for correlations among these values.

2. Methodology

2.1. Hill shading of terrain

Each of the sky models was analytically defined by its radiance (Kijaya, 1986) in every (azimuth, inclination) direction. We used a computer algorithm that analyzed the pattern of sky radiance to choose a representative subset of 250 directions. For that subset of directions, the Voronoi subdivision of the sky dome was computed, resulting in a Voronoi region around each direction. Each direction was weighted by the solid angle subtended by its Voronoi region.

To illuminate the terrain (Figures 1 and 2), we summed 250 point-source renderings of the terrain, one for each of the sampled directions. Each rendering had the point source located in the sampled direction and was weighted by the solid angle associated with the direction. Our experiments showed that 250 samples was sufficient to reliably compute the illumination under any of the sky models (Stewart and Kennelly, 2010).

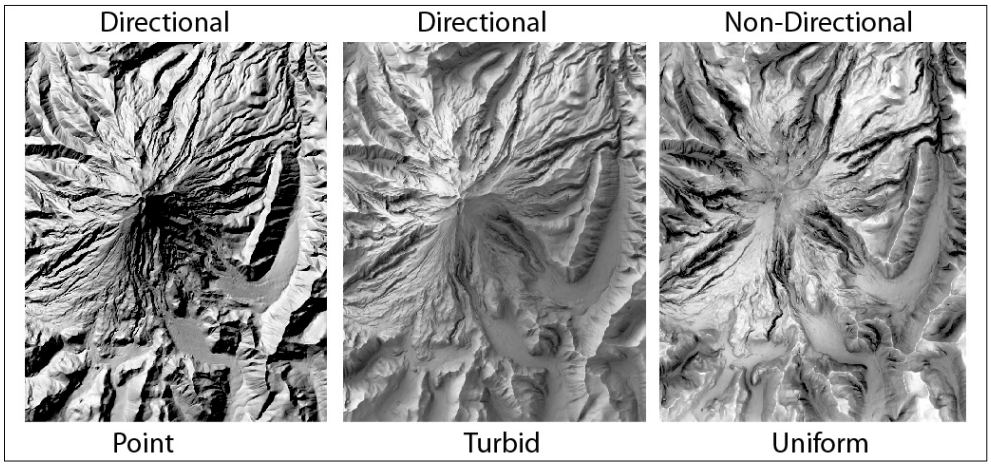

Figure 1. Three hill shadings of Mt. Hood, Oregon.

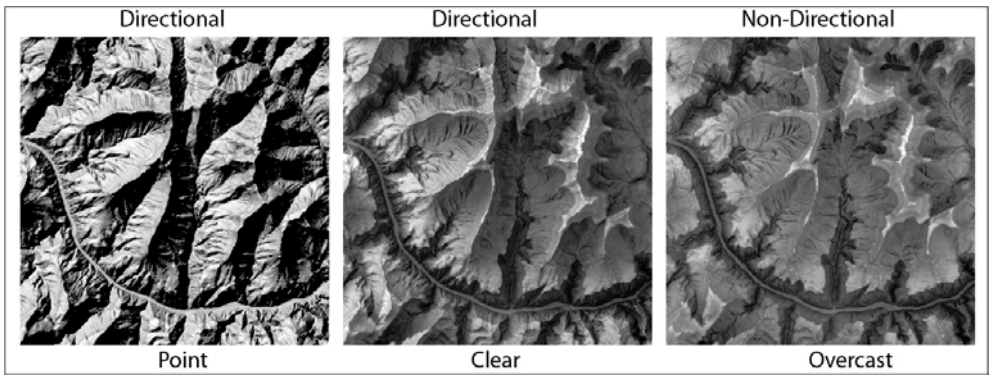

Figure 2. Three hill shadings of the Grand Canyon, Arizona.

2.2. Sampling of hill shaded maps

For each of the study areas shown in Figure 1 and 2, we generated 10,000 random sample points. The attribute tables of all of these points were then populated with values of elevation, as well as values of slope and aspect difference derived from the original digital elevation model (DEM). For directional illumination, this aspect difference is the difference in angle between the aspect direction and the general direction of the illumination source. As a result, aspect difference values vary from 0 to 180 degrees.

Furthermore, the attribute table for the random points was populated with values of grayscale used for shading created with various sky models, as illustrated in Figure 1 and 2. In all cases, a bilinear approximation was used to approximate values from a grid to a point.

2.3. Comparing grayscale values to elevation and derivative values

All points were then used to create 18 scatter plots for each area to compare elevation, slope or aspect difference values with grayscales of any of the sky models described in Section 1. Lower values of grayscale indicate that grid cells are lighter shades of gray, and higher values indicate darker shades of gray. In all cases, a linear regression was calculated on the resulting scatter plots.

The slope of the resulting line can provide insight into the relationship of the two values plotted. For example, a map showing slope shading would be expected to have an associated linear regression slope of -1, because a slope increases grayscale values decrease proportionally. The coefficient of determination, reported here as “R^2”, is an indication of how well the two values correlate.

3. Results

Results of the scatter plots and linear regression are summarized in Table 1 for Mt. Hood and in Table 2 for the Grand Canyon.

From Table 1 (Mt. Hood), scatter plots comparing elevation with point source illumination and uniform sky are presented as Figure 3. Scatter plots comparing aspect difference with point source illumination and turbid sky are presented as Figure 4. In both cases these can be compared with the rendered maps presented in Figure 1.

From Table 2 (Grand Canyon), scatter plots comparing elevation with point source illumination and clear sky are presented as Figure 5. Scatter plots comparing slope with point source illumination and overcast sky are presented as Figure 6. In both cases these can be compared with the rendered maps presented in Figure 2.

Mt. Hood	Elevation	Slope	Aspect Difference
Point	m=-3.8701 R^2=0.03132	m=-0.0286 R^2=0.05873	m=-0.5072 R^2=0.69441
Sharp	m=-2.5313 R^2=0.00236	m=-0.1044 R^2=0.13836	m=0.8966 R^2=0.38224
Clear Sky	m=2.3461 R^2=0.00156	m=-0.0935 R^2=0.08534	m=-1.0026 R^2=0.36791
Turbid Sky	m=4.3960 R^2=0.00604	m=-0.0532 R^2=0.03052	m=-1.0300 R^2=0.42782
Overcast	m=2.0234 R^2=0.00073	m=-0.2942 R^2=0.52994	m=0.1784 R^2=0.00730
Uniform	m=13.8290 R^2=0.02715	m=-0.2622 R^2=0.33611	m=0.1423 R^2=0.00371

Table 1. The slope (m) and coefficient of determination (R^2) for a linear regression of 10,000 random sample points comparing various hill shading models to the elevation, slope and aspect near Mt. Hood, Oregon (Figure 1).

Grand Canyon	Elevation	Slope	Aspect Difference

Point	m=-0.6517 R^2=0.00189	m=-0.0388 R^2=0.04274	m=-0.0836 R^2=0.01878
Sharp	m=9.5377 R^2=0.05429	m=-0.1395 R^2=0.07414	m=-0.1266 R^2=0.00578
Clear Sky	m=9.7655 R^2=0.05161	m=-0.1697 R^2=0.09952	m=-0.1326 R^2=0.00575
Turbid Sky	m=12.2990 R^2=0.0683	m=-0.1089 R^2=0.03416	m=-0.1478 R^2=0.00596
Overcast	m=18.0840 R^2=0.09770	m=-0.3644 R^2=0.25320	m=-0.0190 R^2=0.00006
Uniform	m=24.754 R^2=0.14536	m=-0.3329 R^2=0.16778	m=-0.0328 R^2=0.00015

Table 2. The slope (m) and coefficient of determination (R^2) for a linear regression of 10,000 random sample points comparing various hill shading models to the elevation, slope and aspect near the Grand Canyon, Arizona (Figure 2).

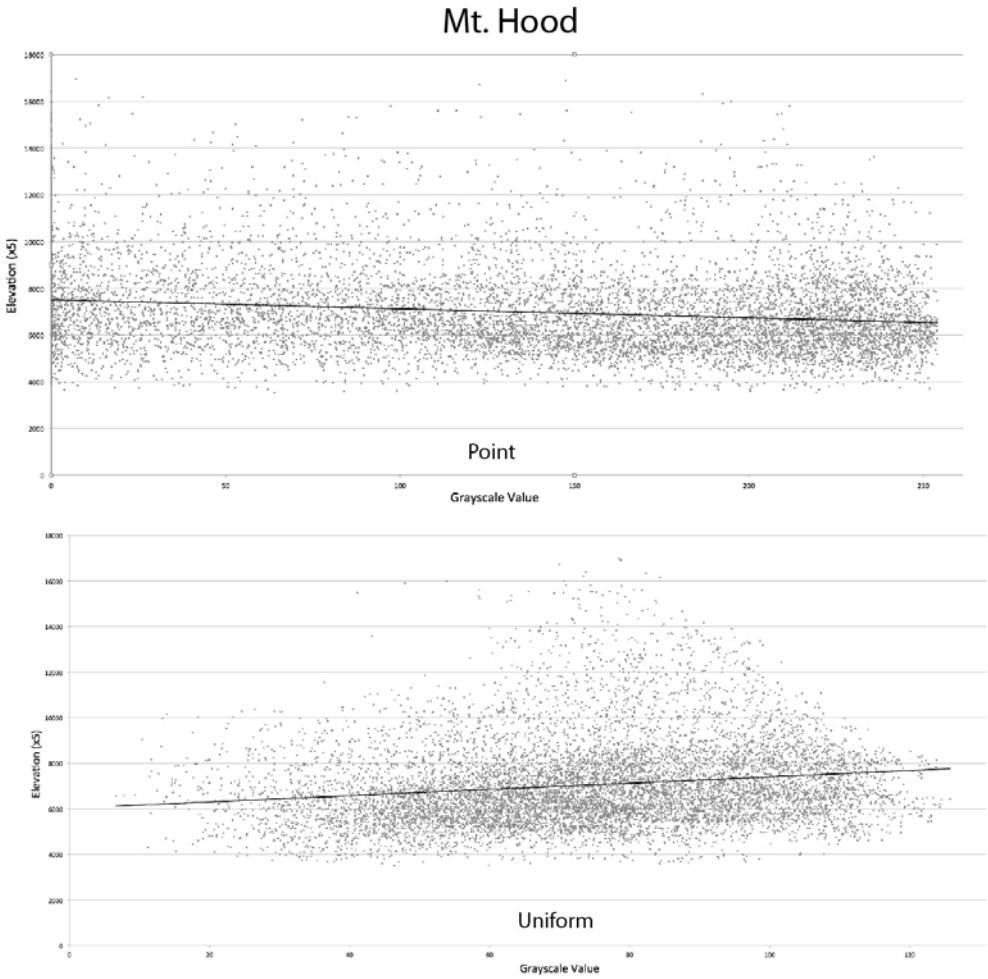

Figure 3. Scatter plots of elevation (vertical exaggeration = 5x's) and grayscale values for 10,000 random points distributed across the maps shown in Figure 1. The linear regression is shown as a dark line, and values are reported in Table 1.

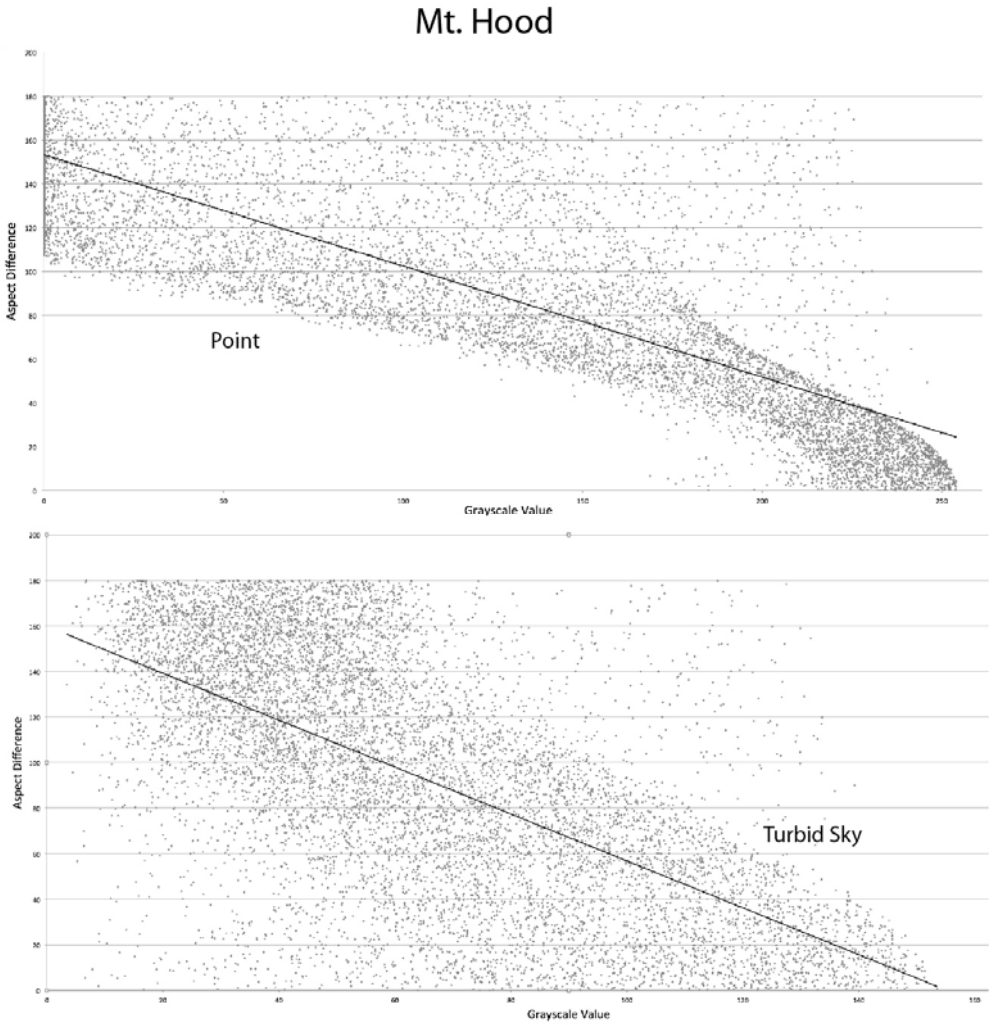

Figure 4. Scatter plots of aspect difference (difference in aspect angle between surface element and the general illumination direction) and grayscale values for 10,000 random points distributed across the maps shown in Figure 1. The linear regression is shown as a dark line, and values are reported in Table 1.

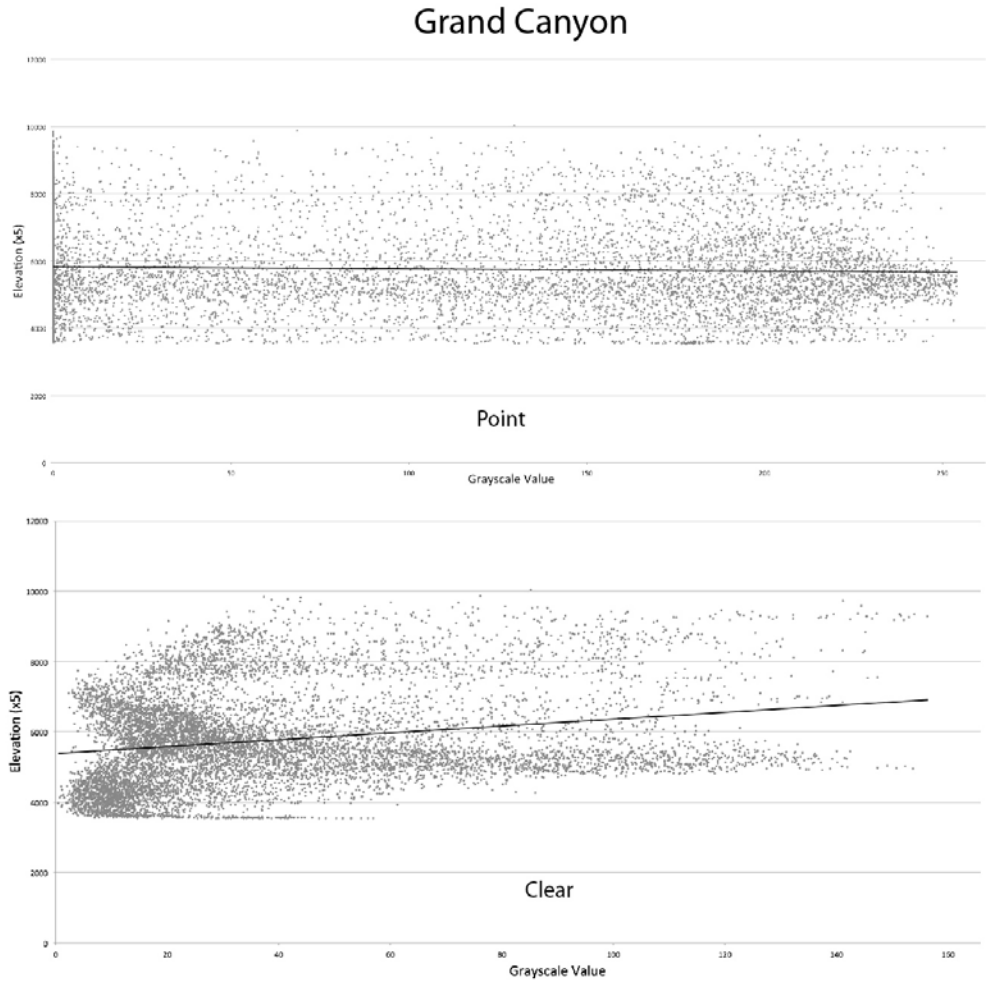

Figure 5. Scatter plots of elevation (vertical exaggeration = 5x's) and grayscale values for 10,000 random points distributed across the maps shown in Figure 2. The linear regression is shown as a dark line, and values are reported in Table 2.

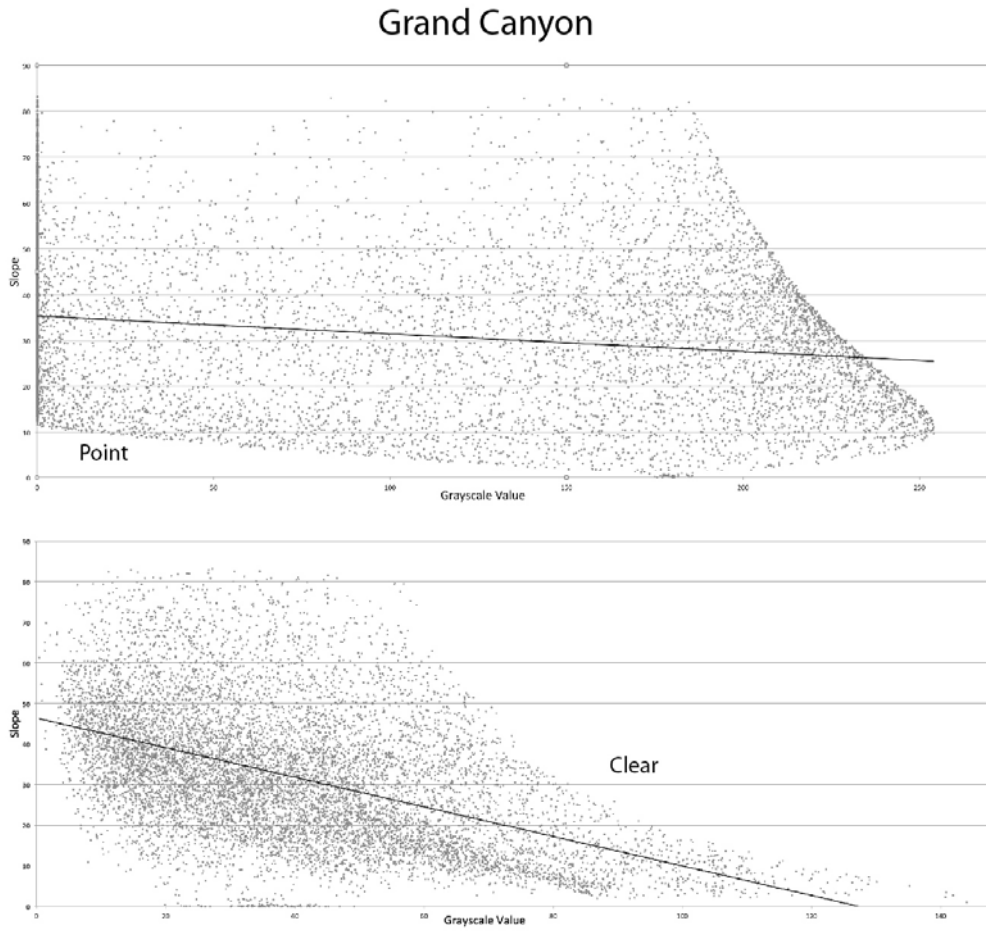

Figure 6. Scatter plots of slope and grayscale values for 10,000 random points distributed across the maps shown in Figure 2. The linear regression is shown as a dark line, with values reported in Table 2.

4. Discussion

The point and uniform hill shadings of Mt. Hood do not show correlation of grayscale values with elevation (Figure 3). This is not unexpected, as hill shading values are associated with the orientation of terrain elements, as opposed to the elevation of terrain. Nevertheless, the uniform sky does have a lack of lower values of shading at higher elevations. This results from areas of higher elevation being exposed to more visible sky, a factor that is more fully accounted for by more complex sky models.

The point and turbid sky hill shadings of Mt. Hood both show a negative correlation to aspect difference values (Figure 4). This seems intuitive for directional illumination, as terrain facing towards the illumination direction will tend to be brighter, and terrain facing away will tend to be darker. The point source scatter plot seems to show some clustering of points at high and low aspect difference angles, while the distribution of points seems smoother across values for the turbid sky rendering.

For the Grand Canyon, neither the grayscale values of the point source or the clear sky rendering tend to correlate well with elevation (Figure 5). Nevertheless, the overall distributions of points in these two scatter plots have strikingly different patterns. The clear sky scatter plot seems to have ranges of elevation within which points lighten or darken with depth. We attribute this to the geometry of the canyon and the resulting amount of the sky that would be blocked from view with increasing depth. A good illustration of this terrain effect is the horizontal trend at the bottom of this scatter plot. This represents the Colorado River channel, from which a greater portion of the sky is sometimes visible.

Both the grayscale values of point and overcast sky rendering show a negative correlation with slope. In the example using point illumination, lower values of slope and higher values of shading both seem to show limits of values associated with the calculation on which the hill shading is based. In the overcast sky scatter plot, such sharp delineations seem to be absent, as a result of illumination originating from a number of point sources clustered around a vector representing the direction of illumination.

5. Conclusion

Hill shading terrain using a point source illumination model is an effective method to represent surface elements with shades of gray in a manner that map users tend to understand (e.g. Robinson et al., 1995; Thelon and Pike, 1991). By utilizing more complex illumination sources such as those adopted by the CIE (e.g. CIE, 2004; CIE 1955), however, terrain or other three dimensional elements can be rendered for map display in a manner which helps the map user understand the geometry of these features (e.g. Kennelly and Stewart, 2006; Stewart and Kennelly, 2010).

The effects of lighting on rendered terrain have been studied by previous cartographic researchers (e.g. Brassel, 1974; Brassel et al., 1974; Horn, 1982; Hugli, 1979; Moellering and Kimerling, 1990; Peucker et al., 1974; Weibel and Heller, 1991) . Some examples include methodologies for locally changing the direction of illumination (Imhof, 1982; Jenny, 2001), and us-

ing various colors of light (Hobbs, 1999). This research differs in using well-defined sky models to illuminate terrain maps.

By using scatter plots to look at correlations among hill shades of gray resulting from various sky models and elevation, slope, and aspect difference, we are able to identify trends and patterns. Elevation does not correlate well with hill shading values from point source rendering, and no patterns are evident in the scatter plot. Elevation also does not correlate well with hill shades of gray from directional or non-directional illumination, but patterns in the scatter plot may help users to interpret such maps. For example, the highest areas tend to appear brighter such as ridges around Mt. Hood, and the deepest areas can appear brighter if more sky is visible, such as the Colorado River channel.

Aspect direction correlates well with both point source and directional sky models for Mt. Hood. This is because terrain elements facing the direction of illumination will generally be brighter, and those facing away will be darker. This results in the perceptual relief effect (Moellering, 2012; Moellering, 2000). Without non-directional shading, no correlation between aspect difference and values of hill shading exist and this three-dimensional effect is found lacking.

Slope does not correlate well with hill shading values from any sky models, but tends to correlate better to grayscale values of non-directional illumination than directional. This may be due to the symmetry of the non-directional illumination with respect to a vertical illumination vector and the associated vertical surface normal vector of a horizontal surface.

As it becomes easier and more common to utilize sky models to create hill shaded maps, general guidelines for which of the gamut of sky models is most useful for various types of terrains will become an important consideration. This study addresses this concern by looking at correlation and patterns of grayscale values in hill shaded maps created with a number of sky models to the terrain's elevation, slope and aspect.

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